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Efficient control charting methodology based on Distance Weighted Mean for normal distribution

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Abstract

This research suggests a Distance Weighted Mean (DWM) based control chart under normal distribution implementing Simple Random Sampling (SRS). The control limits are calculated using the quantile point method. The control chart's performance is assessed using the Average Run Length (ARL) statistic. The numerical findings are illustrated using samples of sizes 3 and 5. The ARL1 values are determined using Monte Carlo Simulation for increasing and decreasing shifts in the location parameter ranging from 5% to 30%. Using the ARL1 measurement, the proposed DWM control charts are compared to the existing Shewhart control charts. According to the comparison analysis, the suggested DWM control chart surpasses the competing Shewhart control chart. The real-life application of the proposed DWM control chart is also shown by using the lifetime of the light bulb (in hours). The results suggest that the proposed DWM control chart can be a useful tool for monitoring process mean shifts, especially when the sample size is large, and the magnitude of the shift is significant.

Keywords: Average Run Length (ARL), Simple Random Sampling (SRS), Distance Weighted Mean (DWM), Monte Carlo Simulation, Shewhart.

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1. Introduction

Control charting was invented by Shewhart (1924). Control charts are frequently used by practitioners to assess one or more quality parameters that are directly or indirectly related to a manufacturing process. Shewhart used well-prepared experiments to provide the framework for the control chart and the idea of statistical control. A control chart is a graph that shows how a process evolves over time. Variable control charts and attribute control charts are the two basic types of control charts. The variable control chart is used to analyse changes in a method using data measured on a continuous scale; it also includes the R-Chart, S-Chart, Exponentially Weighted Moving Average (EWMA) Chart, and Cumulative Sum (CUSUM) Chart. The attribute control chart is based on a discrete measurement scale. The average run length is the number of samples taken before an out-of-control signal is indicated. The control chart with the lowest ARL_1 value is one of the best choices for detecting assignable causes (Montgomery, 2009)."

Control charts have a wide range of applications in the industry for monitoring and controlling process variables to ensure product quality and reduce waste. Control charts are often used in the manufacturing industry to monitor process variables such as temperature, pressure, and flow rates, as well as product attributes such as dimensions and surface finish. The control charts are also used in a variety of different sectors to monitor and regulate process variables in order to enhance quality and efficiency. Control charts are essential tool for many sectors because they can help in identifying process variability, prevent waste, and enhance efficiency.

This article aims to introduce Distance Weighted Mean (DWM) control chart using Simple Random Sampling (SRS) and to further monitor the early detection of the out-of-control signals for the normally distributed lifetime by using Average Run Length (ARL) measure. The study also aims to compare the efficiency of proposed DWM control chart with Shewhart $\bar{X}$ control chart.

1.1. Distance Weighted Mean (DWM)

The DWM is a type of weighted average that calculates the weighting factor for each data point as the inverse of the sum of the distances between those data points and other data points. As a result, the core observations in the data set are given the most weight, while the values at the tails of the distribution are given the least. In other words, data points near other data points are weighted more heavily than isolated data points.

1.2. Normal distribution and applications

The normal distribution, also known as the Gaussian distribution, is a probability distribution that is widely used in statistics to describe real-world phenomena that tend to cluster around a central value. The normal distribution is bell-shaped and symmetric around the mean, with most of the data falling within a certain range of standard deviations from the mean.

Normal distribution is a very effective tool for comprehending and analysing events in life. Its applications are numerous and diverse, and it is a fundamental idea for any individual interested in statistics. As a result, for illustrative and application purposes, samples generated by the normal distribution are used in this study.

2. Literature review

Aczel (1989) argued that once the interruptions are gone, a process's capacity is its regular behaviour. According to Montgomery (2009), the processes which perform in the presence of assignable or special causes may be classified as out of control processes. The control charts, according to Abbas *et al.* (2012), are the most commonly used technique for detecting the presence of "special causes of process changes." Memory control charts and memory-less control charts are the two forms of control charts. Memory control charts, as opposed to memoryless control charts such as Shewhart type control charts, are designed such that past data is not lost. Butt and Raza (2017) explained the performance of the Exponentially Weighted Moving Average (EWMA) control chart in the presence of Type I censored data. Chen *et al.* (2004) developed a new control chart based on the EWMA technique. The control region was defined by the statistic for the chart as the area below a straight line, which simplifies the charting method.

Grant and Leavenworth (1980) described Statistical Process Control (SPC) as a valuable and significant technique used regularly in the manufacturing industry to monitor the whole process. SPC may be used in a variety of engineering situations. The process will become more consistent and dependable as a result of the extensive use of SPC analysis. Gupta and Gupta (2009) described statistical quality control as one of the most beneficial and cost-effective use of sampling theory in the industrial area. Castagliola *et al.* (2015) recommended a different methodology for monitoring the coefficient of variation for a finite horizon output using one-sided Shewhart-type charts. Raji *et al.* (2018) suggested Tukey and median absolute deviation outlier detectors as models for identifying Phase-I errors. They described that "both of these outlier detection models are efficient, robust, and distribution-free. The findings show that when outlier detectors are available, the suggested design structures are more stable and require less Phase-I observation to stabilize the run-length characteristics. Finally, they apply the findings of a current study to the semiconductor manufacturing industry, where an actual dataset is retrieved from a photo-lithography process."

Woodall (2006) shows the use of control charts in healthcare and public-health surveillance. They discussed the use of control charts in healthcare and public-health surveillance, including the different types of control charts that can be used and their advantages and limitations. Roberts (2000) proposed control chart tests based on Geometric Moving Averages (GMA) concept, which is a type of control chart used for the monitoring of the non-normal data. Riaz (2008) proposed a Q-chart for monitoring the process dispersion. Ali *et al.* (2021) presented deviation based EWMA control charts to monitor type-I censored data. The primary goal of their research was to present novel control charting techniques based on the absolute differences of moving averages of means under the EWMA and DEWMA design structures. Hyder *et al.* (2022) proposed EWMA-MSS(S) and CUSUM-MSS(S) charts under MSS schemes to monitor moderate to small shifts in the location parameter of a process.

3. Methodology

3.1. DWM based control chart for normal distribution

In this section, the structure of the DWM control chart is introduced. Consider dataset $t_1, t_2, \ldots, t_n$ observations and $L_1, L_2, \ldots, L_n$ are weighted coefficients.

Distance Weighted Mean (DWM) $= \dfrac{L_1 t_1 + L_2 t_2 + L_3 t_3 + ... + L_n t_n}{L_1 + L_2 + L_3 + ... + L_n}$ (1)

The weighting coefficient for t_i is computed as the inverse mean distance between t_i and the other data points.

$$\text{Distance Weighted Mean} = \frac{\sum_{i=1}^{n} L_i t_i}{\sum_{i=1}^{n} L_i} \quad (2)$$

Where,

$$L_i = \frac{r}{\sum_{i=1}^{n} \left| t_i - t_j \right|} \quad (3)$$

Where, r shows some positive quantity. The most choices of 'r' in the literature is 1 or n-1. Also, "j" shows the sample units of each iteration (Dodonov & Dodonova, 2011).

The normal distribution is used to monitor the performance of average run length-out of control (ARL_1). The normal distribution pdf is defined as:

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} \quad (4)$$

Where, scale parameter is σ and location parameter is μ.

The SDRL is the standard deviation of the run length, and it measures the spread of the run length values from their ARL. As the magnitude of shifts increases, the standard deviation of run length decreases.

3.2. Construction of DWM control chart

In this section, the construction of distance weighted mean-based control chart by using Monte Carlo Simulation is described. It involves the following steps:

Step 1:
200 samples of size 3 and 5 are generated by using random numbers from a normal distribution with $\mu = 5, \sigma = 1.25, \mu = 10, \sigma = 2.5$ and $\mu = 15, \sigma = 4$ respectively.

Step 2:
DWM was computed for each of the 200 samples obtained in equation (1).

Step 3:
Expected value of DWM was computed for 200 samples obtained in equation (2).

Step 4:
Standard deviation of DWM for 200 samples (equation 1) computed.

Step 5:
By using 99.73% quantile point approach, control limits were computed using results obtained in equation (3) under normal distribution.

Step 6:
Increasing and decreasing shifts in the location parameter were introduced in next 200 samples.

Step 7:
DWM, expected value of DWM and standard deviation of DWM was calculated for the 200 out-of-control samples as obtained in equations (2), (3) and (4).

Step 8:
The charting statistics were plotted against the control limits.

Step 9:
When the charting statistics fall outside the control limits the number is recorded as Run Length (RL).

Step 10:
The steps from 1 to 9 were repeated 10000 times and the 10000 RLs are obtained.

Step 11:
Then the average of RLs and Standard Deviation (SD) of RLs is taken to obtain the ARL_1 and SDRL values.

Step 12:
The whole procedure from step 1 to 11 is repeated for 370 and 500 samples of size 3 and 5.

Step 13:
The comparison was made by using average run length and SDRL measure. The ARL_1 curves were used to compare the efficiency of the proposed Distance Weighted Mean control chart with $\bar{X}$ control chart.

4. Results and discussion

In this section, the findings of the proposed DWM based control chart for normal distribution are described and interpreted. The sample size 3 and 5 were used for the evaluation. For the detection of false alarm rate Average Run Length (ARL) values were computed for different increase and decrease shifts in the location parameter. Furthermore, ARL_1 curve is made that shows a graphical view of exploring the average run length performance. The DWM control chart is also computed with $\bar{X}$ control chart.

Table-1 shows the ARL_1 and SDRL values for DWM and $\bar{X}$ control charts for n = 3 and 5, using normal distributions with parameter $\mu = 5$ and $\sigma = 1.25$, along with an ARL_0 value of 200. The values for ARL_1 (Average Run Length of the chart to signal a shift) and SDRL

(Standard Deviation of the Run Length) have been calculated for increasing and decreasing shifts in the location parameter of the distribution. It is observed that for both increasing and decreasing shifts, the DWM control chart outperforms the $\overline{X}$ control chart, particularly for larger sample sizes (n = 5).

Table-1: ARL_1 and SDRL values for DWM and $\overline{X}$ control chart for n = 3,5 with specified parameters N(5,1.25) using ARL_0 = 200

N = 3					
Shifts in location parameter (%)		DWM Chart		$\bar{X}$ Chart	
		ARL_1	SDRL	ARL_1	SDRL
5	Increasing	105.2	163.0	159.7	186.6
10		63.80	153.2	80.88	161.0
15		22.65	65.55	59.39	70.19
20		9.945	14.98	19.65	32.46
25		5.165	7.863	9.761	11.50
30		2.972	6.091	4.792	5.861
5	Decreasing	118.0	182.6	158.8	192.0
10		63.98	156.4	126.1	169.4
15		25.35	77.74	53.21	108.6
20		11.57	38.51	19.07	21.32
25		5.316	7.842	9.192	11.37
30		2.858	4.212	4.834	5.517
n = 5					
Shifts in location parameter (%)		DWM Chart		$\bar{X}$ Chart	
		ARL_1	SDRL	ARL_1	SDRL
5	Increasing	95.51	127.9	117.7	150.6
10		33.77	89.72	59.91	107.8
15		12.84	30.40	21.06	34.56
20		4.585	7.493	8.879	11.38
25		2.370	3.260	3.666	4.614
30		1.195	1.764	1.650	2.228
5	Decreasing	90.46	185.5	100.9	168.6
10		37.01	95.96	87.76	149.5
15		13.89	46.58	20.62	33.29
20		4.579	6.723	7.812	8.962
25		2.177	3.340	3.665	4.443
30		1.129	1.852	1.767	2.330

For example, for a 5% increasing shift in the location parameter and n = 5, the ARL1 value for the DWM chart is 95.51 with SDRL = 127.9, while for the control chart it is 117.7 with SDRL = 150.6. Similarly, for a 10% decreasing shift in the location parameter and n = 5, the ARL1 value for the DWM chart is 37.01 with SDRL = 95.96, while for the control chart it is 87.76 with SDRL = 149.5.

Overall, the results suggest that the DWM control chart is more effective in detecting shifts in the location parameter of the distribution, particularly for larger sample sizes.

From Table-2 it is observed that samples from N(5,1.25) using ARLo = 370, the ARL_1 values for DWM chart is less than the Shewhart $\overline{X}$ control chart therefore the DWM chart outperforms the $\overline{X}$ control chart.

Table-2: ARL_1 and SDRL values for DWM and $\overline{X}$ control chart for n = 3,5 with specified parameters N(5,1.25) using ARL_o = 370

n = 3					
Shifts in location parameter (%)		DWM Chart		$\bar{X}$ Chart	
		ARL_1	SDRL	ARL_1	SDRL
5	Increasing	134.2	205.1	327.9	279.2
10		61.17	123.2	159.1	218.7
15		25.93	65.31	47.21	82.08
20		18.95	19.60	18.95	19.60
25		5.879	8.792	8.244	9.404
30		2.926	3.700	4.804	5.420
5	Decreasing	128.9	199.2	325.6	278.2
10		64.11	133.1	155.0	218.8
15		23.17	46.30	50.86	99.09
20		11.77	24.98	19.41	28.08
25		5.527	7.766	8.940	10.43
30		3.014	4.688	4.806	5.478
n = 5					
Shifts in location parameter (%)		DWM Chart		$\bar{X}$ Chart	
		ARL_1	SDRL	ARL_1	SDRL
5	Increasing	107.8	183.9	288.7	275.1
10		35.50	83.54	74.47	133.4
15		12.47	24.82	20.32	28.06
20		5.083	7.960	8.086	8.964
25		2.554	3.740	3.481	4.101
30		1.207	1.791	1.688	2.048
5	Decreasing	117.5	189.1	277.2	273.0
10		36.46	79.81	84.84	147.0
15		14.26	42.31	21.20	30.20
20		5.402	7.668	7.997	9.248
25		2.408	3.610	3.578	4.125
30		1.328	2.092	1.720	2.128

Similarly, from Table-3 it is observed that samples from N(5,1.25) using ARLo = 500, The ARL_1 values for DWM chart is less than the Shewhart $\overline{X}$ control chart therefore, the DWM chart outperforms the $\overline{X}$ control chart.

Table-3: ARL_1 and SDRL values for DWM and $\overline{X}$ control chart for n = 3 with specified parameters N(5,1.25) using ARL_o = 500

n = 3					
Shifts in location parameter (%)		DWM Chart		$\bar{X}$ Chart	
		ARL_1	SDRL	ARL_1	SDRL
5	Increasing	114.4	152.1	255.0	209.3
10		57.70	98.75	119.7	153.2
15		25.56	43.23	43.83	61.58
20		11.80	20.39	18.29	18.79
25		5.623	7.153	8.876	10.61
30		3.156	3.834	4.731	5.693
5	Decreasing	121.8	156.2	267.0	210.8
10		57.20	94.71	126.2	157.1
15		24.67	42.65	41.98	56.90
20		11.37	13.92	18.31	19.44
25		6.020	8.139	8.907	9.691
30		3.412	5.019	4.648	5.232
n = 5					
Shifts in location parameter (%)		DWM Chart		$\bar{X}$ Chart	
		ARL_1	SDRL	ARL_1	SDRL
5	Increasing	101.5	142.2	217.1	205.1
10		36.62	66.46	67.32	98.55
15		12.64	16.57	19.08	20.45
20		5.677	7.172	7.835	8.792
25		2.528	3.413	3.536	4.196
30		1.223	1.763	1.660	2.014
5	Decreasing	99.53	142.9	221.7	207.3
10		35.02	57.34	66.53	93.58
15		12.83	16.09	20.52	27.05
20		5.559	7.351	7.913	8.526
25		2.653	3.347	3.716	4.719
30		1.301	1.937	1.594	2.063

From Figures 1 and 2, it is observed that with the increasing and decreasing shifts (5% to 30%) in the location parameter the ARL_1 values for DWM charts are less than the $\bar{X}$ chart when n = 3, $\mu = 5$, $\sigma = 1.25$ and $ARLo = 370$.

Figure 1: ARL_1 curves of DWM and $\bar{X}$ control chart for n = 3 with μ = 5,σ = 1.25 and ARL_o = 370

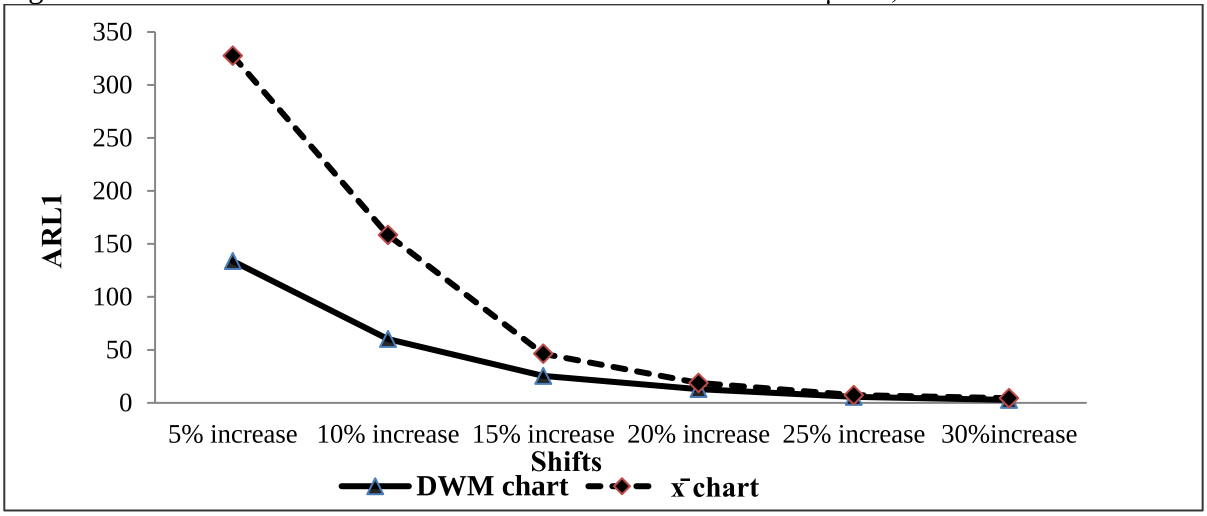


Figure 2: ARL_1 curves of DWM and $\bar{X}$ control chart for n = 3 with μ = 5, σ = 1.25 and ARL_o = 370

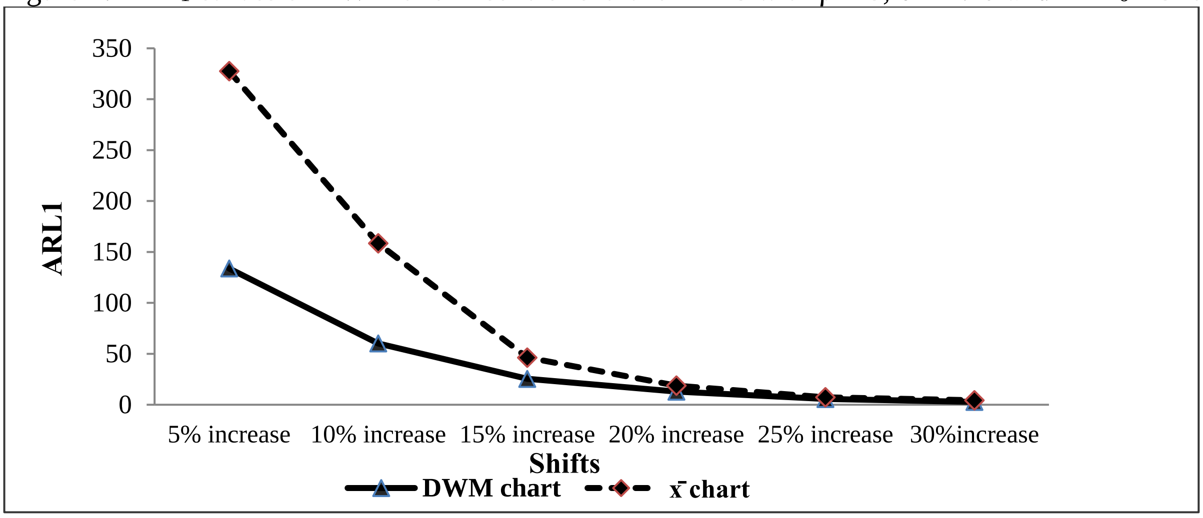
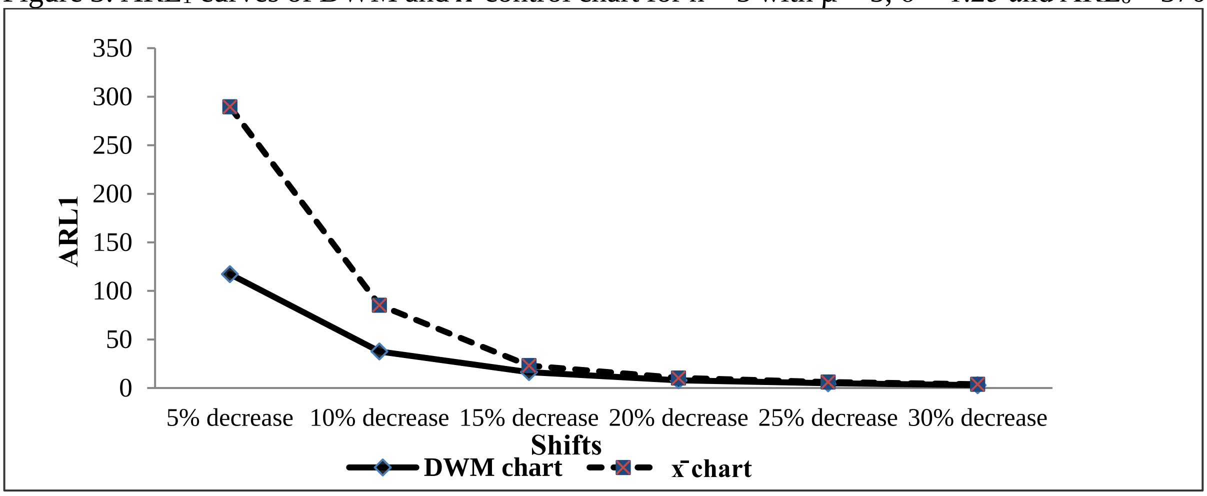
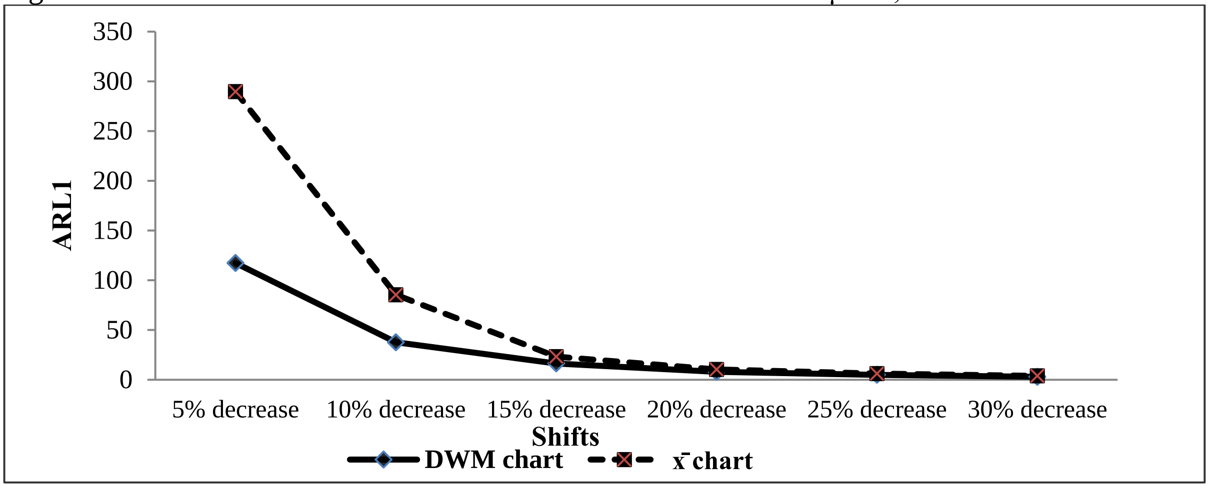


Figure 3: ARL_1 curves of DWM and $\bar{X}$ control chart for n = 5 with μ = 5, σ = 1.25 and ARL_o = 370



Figures 4: ARL_1 curves of DWM and $\bar{X}$ control chart for n = 5 with μ = 5,σ = 1.25 and ARL_0 = 370



From Figures 3 and 4 it is observed that with the increasing and decreasing shifts (5% to 30%) in the location parameter the ARL_1 values for DWM chart are less than the $\bar{X}$ chart when n = 5, μ = 5, σ = 1.25 and ARLo = 370. It can be seen that when we increased the sample size, it gave smaller ARL_1 values.

5. Application

In this section, real data example to show how the proposed methodology can be used in practical situations is described.

5.1. Bulb life time (hours) data

To investigate the use of the suggested DWM control chart, light bulb lifetime data in hours is used. The data set included a sample of 40 light bulbs. Easy fit is used to examine the data distribution and discovered that the bulb lifespan (hours) data followed a normal distribution. The 40 random samples of size 3 were chosen using replacement. There are 20 samples used for phase I monitoring and 20 samples used for phase II monitoring. We incorporated a 20% rising shift in the mean bulb lifetime (hours) for phase-II monitoring. The UCL was predicted to be 1392.33 and the LCL to be 1109.05.

Figure 5: Application of DWM control chart on real data

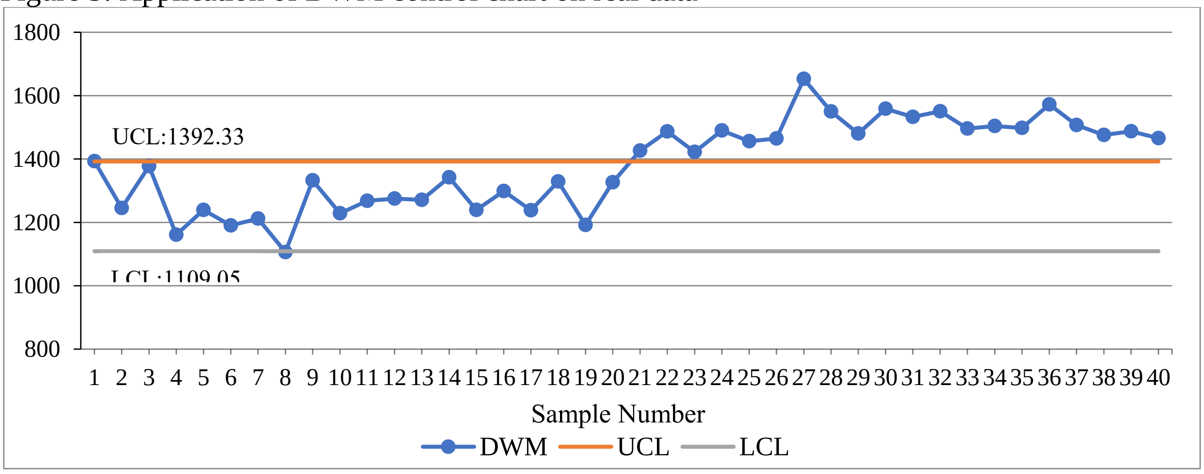


Figure 5 shows the DWM control chart giving the first out-of-control at the 21 sample number. We observed that the ARL1 value was 1.140. That is, for every 20% increase in the location parameter, we require 1 sample to detect the first out-of-control signal. In other words, the observer writes 14000 hours of bulb life instead of 1400 hours at the time of recording. This inaccurate reading causes a significant shift in the experiment and leads to an incorrect assessment of the bulb lifetime. As a result, in this case, the DWM control chart gives less weight to the incorrect observation that was acting as an outlier. That is why, it gives an early detection of out-of-control signal and enhance the efficiency of proposed DWM control chart.

6. Conclusion and recommendations

In this article, the authors proposed a new control chart called the “Distance Weighted Mean (DWM) based control chart” and compared it with the traditional Shewhart control chart. The comparison was based on the Average Run Length (ARL_1) values obtained through Monte Carlo simulation, using increasing and decreasing shifts in the location parameter ranging from 5% to 30%.

The results showed that the proposed DWM control chart outperformed the Shewhart control chart for both increasing and decreasing shifts in the location parameter. Specifically, the ARL_1 values for the DWM control chart were found to be consistently lower than those of the Shewhart control chart, indicating that the DWM control chart was more efficient in detecting shifts in the process mean.

Furthermore, the authors observed that increasing the sample size led to lower ARL_1 values for the DWM control chart, indicating improved performance. Similarly, increasing the magnitude of the shift also led to lower ARL_1 values for the DWM control chart, indicating increased efficiency.

Overall, the results suggest that the proposed DWM control chart can be a useful tool for monitoring process mean shifts, especially in situations where the sample size is large and the magnitude of the shift is significant. This work can be further extended for Cumulative Sum (CUSUM) and Mixed EWMA-CUSUM(MEC) control chart.

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Appendix:

Table: DWM Statistics and control limits calculation process

Samples			w1	w2	w3	DWM	UCL	LCL	DWM BAR	SD BAR	UCL	LCL
5.675529	5.10485	5.250828	1.00464	1.395367	1.752297	5.304528	6.980597	3.010091	4.99018264	0.734534	6.980597	3.010091
4.306671	3.911458	5.974296	0.484769	0.406826	0.268063	4.553507	6.980597	3.010091				
5.811056	4.351492	5.580828	0.591789	0.371899	0.685136	5.386179	6.980597	3.010091				
3.765854	5.837349	5.943186	0.235359	0.459278	0.437988	5.44782	6.980597	3.010091				
4.856204	6.08718	3.066713	0.331075	0.235214	0.207902	4.749646	6.980597	3.010091				
2.25401	4.541966	3.959618	0.250403	0.348395	0.437071	3.74318	6.980597	3.010091				
3.721541	5.88579	3.704357	0.458414	0.230113	0.454831	4.150284	6.980597	3.010091				
7.142865	4.803882	6.118206	0.297297	0.273725	0.427536	6.062991	6.980597	3.010091				
5.671357	3.11572	4.863337	0.297296	0.232382	0.391292	4.683207	6.980597	3.010091				
5.68822	6.833511	3.07235	0.265875	0.203813	0.156813	5.406056	6.980597	3.010091				
5.428717	2.88539	5.094755	0.347549	0.210407	0.393186	4.728041	6.980597	3.010091				
6.021958	4.421367	4.728521	0.345539	0.524179	0.624769	4.919843	6.980597	3.010091				
3.321292	4.616851	3.226516	0.719251	0.372316	0.673351	3.558436	6.980597	3.010091				
6.365204	5.873771	4.885739	0.507383	0.67592	0.405269	5.778671	6.980597	3.010091				
6.633603	6.01011	4.689272	0.389435	0.514316	0.306263	5.876464	6.980597	3.010091				
3.528413	6.235686	4.596872	0.264849	0.230092	0.369375	4.705742	6.980597	3.010091				
4.880359	3.502575	3.6105	0.377694	0.673079	0.725803	3.839579	6.980597	3.010091				
7.217269	4.975451	3.825802	0.177516	0.294858	0.22021	5.184516	6.980597	3.010091				
4.717599	6.416565	3.981648	0.410692	0.241903	0.315371	4.902407	6.980597	3.010091				
2.920784	4.282614	3.016035	0.686303	0.380458	0.734306	3.247292	6.980597	3.010091				
4.449197	5.709388	5.451159	0.442057	0.658579	0.79353	5.307106	6.980597	3.010091				
7.050713	3.364944	5.103587	0.177529	0.184352	0.271314	5.143304	6.980597	3.010091				
4.153359	4.565328	5.283764	0.648351	0.884639	0.540879	4.623908	6.980597	3.010091				
5.191911	4.482289	5.07797	1.214237	0.766105	1.409201	4.984151	6.980597	3.010091				
3.806668	4.020909	5.541407	0.513089	0.576456	0.307197	4.276625	6.980597	3.010091				
5.91171	6.822834	4.899645	0.51997	0.352819	0.340686	5.89248	6.980597	3.010091				
4.763376	2.985271	3.383203	0.316628	0.459551	0.562396	3.573055	6.980597	3.010091				
5.828828	4.141981	5.292341	0.449775	0.352459	0.592822	5.174671	6.980597	3.010091				
4.823521	5.852225	5.693756	0.52661	0.842337	0.972097	5.555018	6.980597	3.010091				
4.901828	4.245094	4.67986	1.138044	0.91617	1.522687	4.639124	6.980597	3.010091				
6.534007	4.337895	6.423141	0.433468	0.233571	0.45535	6.032015	6.980597	3.010091				
6.624266	3.889244	5.390714	0.25198	0.236044	0.365628	5.339658	6.980597	3.010091				
5.845819	5.827471	4.531777	0.75053	0.76101	0.38318	5.572703	6.980597	3.010091				
5.474306	4.245077	3.312901	0.29493	0.462662	0.32325	4.30171	6.980597	3.010091				
4.388281	4.210381	5.562507	0.739576	0.653583	0.395828	4.583094	6.980597	3.010091				
5.429981	4.276572	4.269004	0.43208	0.861343	0.855765	4.505443	6.980597	3.010091				
4.182783	5.702815	4.985842	0.430461	0.447026	0.657881	4.969442	6.980597	3.010091				
6.191727	3.32326	2.309483	0.148133	0.257583	0.204248	3.680416	6.980597	3.010091				
7.340281	6.166309	2.666247	0.170998	0.213948	0.122338	5.717956	6.980597	3.010091				

4.783173	6.046561	5.281741	0.567551	0.493046	0.791523	5.332563	6.980597	3.010091				
4.424859	4.689727	4.578742	2.388057	2.660621	3.775472	4.570561	6.980597	3.010091				
7.749881	6.008586	4.253907	0.190939	0.286043	0.190452	6.006036	6.980597	3.010091				
7.411178	6.683396	4.334621	0.262858	0.325039	0.184321	6.370499	6.980597	3.010091				
7.725089	6.209489	4.06349	0.193155	0.273105	0.172188	6.089243	6.980597	3.010091				
4.098049	6.063624	3.713837	0.42557	0.23173	0.365765	4.4059	6.980597	3.010091				
5.638004	4.938797	5.641523	1.42303	0.713301	1.415939	5.499005	6.980597	3.010091				
2.261085	7.56727	4.174903	0.138504	0.114962	0.188459	4.457576	6.980597	3.010091				
4.739951	5.920966	4.13286	0.559251	0.3368	0.417502	4.849809	6.980597	3.010091				
4.48649	5.783055	4.679288	0.671428	0.416609	0.771268	4.856983	6.980597	3.010091				
5.106262	2.87988	4.260907	0.325549	0.277207	0.449159	4.158592	6.980597	3.010091				
5.294058	3.442415	4.944924	0.454385	0.298138	0.540061	4.721098	6.980597	3.010091				
4.540502	4.325219	9.057352	0.211321	0.202126	0.10812	5.393408	6.980597	3.010091				
3.974602	4.790526	4.620093	0.684268	1.013831	1.225604	4.528121	6.980597	3.010091				
5.57129	6.376151	6.997416	0.448232	0.7012	0.488427	6.341153	6.980597	3.010091				
4.099452	2.310166	5.280023	0.336717	0.210122	0.240939	3.983274	6.980597	3.010091				
3.540908	7.135742	5.236984	0.189003	0.18203	0.278177	5.275596	6.980597	3.010091				
5.862522	5.468016	7.681953	0.451684	0.38337	0.247932	6.139398	6.980597	3.010091				
3.398433	6.825256	4.511027	0.220293	0.174184	0.291815	4.741258	6.980597	3.010091				
4.951761	3.419356	4.453616	0.492478	0.389611	0.652569	4.3509	6.980597	3.010091				
3.429133	3.844412	6.047054	0.329685	0.381983	0.207445	4.192591	6.980597	3.010091				
5.134808	3.881292	4.129734	0.442754	0.665798	0.797756	4.276399	6.980597	3.010091				
4.288021	3.249454	3.223976	0.475599	0.93981	0.917833	3.45113	6.980597	3.010091				
5.888044	2.889265	5.429319	0.289226	0.180543	0.333469	5.023568	6.980597	3.010091				
4.788309	5.147643	4.329253	1.221912	0.849096	0.782812	4.769301	6.980597	3.010091				
4.715687	7.177859	4.26067	0.342796	0.185896	0.296542	5.106819	6.980597	3.010091				
4.624745	7.194517	5.131934	0.324996	0.215873	0.38914	5.43346	6.980597	3.010091				
6.943164	4.347986	3.497123	0.165529	0.290188	0.232726	4.68434	6.980597	3.010091				
4.98238	5.255177	7.038943	0.429303	0.486248	0.260394	5.550573	6.980597	3.010091				
6.785792	7.407878	5.563556	0.542205	0.405448	0.326099	6.670898	6.980597	3.010091				
7.562797	5.382539	4.025443	0.174898	0.282697	0.204313	5.539737	6.980597	3.010091				
5.264938	4.687761	4.510667	0.751063	1.325785	1.073695	4.765002	6.980597	3.010091				
4.614836	3.495246	4.265051	0.680562	0.52927	0.893184	4.184508	6.980597	3.010091				
3.700408	4.669852	5.335075	0.384008	0.611745	0.434803	4.611809	6.980597	3.010091				
3.759194	4.93724	4.277024	0.589666	0.543992	0.848863	4.304164	6.980597	3.010091				
2.78492	3.771341	4.032262	0.447675	0.801705	0.663014	3.630888	6.980597	3.010091				
5.010563	5.293577	5.470153	1.346612	2.175851	1.571916	5.273251	6.980597	3.010091				
8.120809	3.642459	7.688475	0.203638	0.117311	0.223297	6.97813	6.980597	3.010091				
4.715804	3.594493	5.054084	0.685123	0.387461	0.556213	4.564583	6.980597	3.010091				
4.226575	4.724458	2.628216	0.477044	0.385486	0.270665	4.014173	6.980597	3.010091				
7.27081	5.474219	5.701724	0.297117	0.494048	0.55661	5.964234	6.980597	3.010091				
4.50276	5.180795	5.283002	0.68574	1.281654	1.133211	5.068193	6.980597	3.010091				
4.436983	5.149716	6.669838	0.339491	0.447857	0.266455	5.304468	6.980597	3.010091				
5.424682	5.425356	6.781537	0.736632	0.736998	0.368591	5.69643	6.980597	3.010091				

5.506234	5.02559	3.981359	0.498624	0.655792	0.389241	4.917537	6.980597	3.010091				
5.120021	4.81482	6.598318	0.560696	0.478767	0.30658	5.348168	6.980597	3.010091				
3.884146	4.704049	5.630446	0.389681	0.57264	0.374154	4.724337	6.980597	3.010091				
5.675703	5.263118	6.280864	0.982563	0.699138	0.616178	5.712447	6.980597	3.010091				
3.102341	5.312985	3.249649	0.424097	0.233974	0.452357	3.628147	6.980597	3.010091				
6.174193	5.867236	6.023706	2.186061	2.157838	3.257787	6.022566	6.980597	3.010091				
4.868068	5.558283	5.65735	0.675905	1.266974	1.125683	5.442593	6.980597	3.010091				
4.485746	6.643637	5.735601	0.293449	0.326166	0.463416	5.670415	6.980597	3.010091				
4.990822	4.299593	4.201084	0.675234	1.266242	1.125812	4.415604	6.980597	3.010091				
8.404516	3.733256	6.620417	0.15491	0.132303	0.214075	6.40975	6.980597	3.010091				
5.987398	4.483759	4.050461	0.290649	0.516279	0.421899	4.690642	6.980597	3.010091				
6.335951	4.614545	4.984995	0.325482	0.478044	0.58092	5.174689	6.980597	3.010091				
4.436768	3.513899	4.307046	0.950037	0.582745	1.083578	4.177491	6.980597	3.010091				
4.725103	5.984303	5.663748	0.454991	0.63301	0.794155	5.54465	6.980597	3.010091				
6.751117	5.638598	2.947112	0.203396	0.262881	0.153953	5.335354	6.980597	3.010091				
5.235967	5.42059	3.851448	0.637291	0.570202	0.338563	5.000869	6.980597	3.010091				
5.056468	3.348014	5.407018	0.485672	0.265431	0.415015	4.79235	6.980597	3.010091				
4.756958	5.870974	4.765306	0.890977	0.450514	0.897653	4.984444	6.980597	3.010091				
4.774618	5.135551	6.05502	0.60926	0.781005	0.454572	5.242912	6.980597	3.010091				
5.590122	3.383521	6.119019	0.365564	0.202343	0.306335	5.264731	6.980597	3.010091				
4.433511	4.137805	5.192008	0.948584	0.740791	0.551663	4.522478	6.980597	3.010091				
5.128419	7.403134	5.131443	0.439032	0.219954	0.439616	5.585056	6.980597	3.010091				
5.635756	4.771678	6.071203	0.769512	0.462192	0.576378	5.553687	6.980597	3.010091				
5.930144	6.705011	5.028415	0.596447	0.40792	0.387849	5.905973	6.980597	3.010091				
5.566599	5.342492	6.216722	1.143864	0.910467	0.656016	5.648673	6.980597	3.010091				
4.067284	6.639181	6.351312	0.205934	0.349679	0.388818	5.959862	6.980597	3.010091				
2.497194	5.379185	6.442682	0.146467	0.253454	0.199641	5.029265	6.980597	3.010091				
4.73283	5.13613	4.968132	1.565919	1.750402	2.479544	4.955295	6.980597	3.010091				
4.736906	4.545587	6.724653	0.458912	0.421872	0.239992	5.090527	6.980597	3.010091				
5.587489	3.550359	6.224584	0.37394	0.212253	0.301994	5.317289	6.980597	3.010091				
4.780363	6.510804	7.108225	0.246408	0.429579	0.341847	6.292529	6.980597	3.010091				
3.200147	5.577459	4.586889	0.265671	0.296922	0.420643	4.511328	6.980597	3.010091				
6.040008	3.186081	5.007689	0.257318	0.213879	0.350394	4.856798	6.980597	3.010091				
4.353462	5.642182	4.721985	0.603412	0.452711	0.775964	4.827991	6.980597	3.010091				
5.043052	4.318714	4.688505	0.926883	0.913969	1.380571	4.685601	6.980597	3.010091				
4.22706	4.883839	3.706126	0.849103	0.54511	0.588704	4.252952	6.980597	3.010091				