Natural and Applied Sciences International Journal (NASIJ)
eISSN: 2788-4619 (online)
https://www.ideapublishers.org/index.php/nasij
Vol. 5, No. 2 (July-December), 2024, pp. 32-55
https://doi.org/10.47264/idea.nasij/5.2.3
Research Article | Original Research

Volatility spillover effect between cryptocurrency and stock market using MGARCH BEKK model

Iqra Hussain[1] | Nazakat Ali*[1] | Hafiz Bilal Ahmad[1] | Suhail Ashraf[2]

1. Department of Mathematics and Statistics, University of Agriculture, Faisalabad, Pakistan.
2. School of Mathematics, Computer Science and Engineering, Liverpool Hope University, Liverpool, United Kingdom.

* Corresponding Author Email: nazakatalibandesha@gmail.com

Abstract:

This paper explores the volatility spillover effects between the cryptocurrency market and the Pakistan Stock Exchange (PSX). Utilising data from January 1, 2019, to April 5, 2024, sourced from Investing and Yahoo Finance, the study employs the Multivariate Generalized Autoregressive Conditional Heteroskedasticity (MGARCH) BEKK model to assess the dynamic interactions between these markets. Stationarity tests confirmed the non-stationarity of time series data at their levels, which became stationary after first differencing, ensuring robust econometric analysis. The results indicate significant volatility spillovers from major cryptocurrencies, such as Bitcoin and Ethereum, to the PSX, highlighting a solid interconnectedness between these markets. This suggests that digital asset volatility significantly influences traditional financial systems. The study concludes that integrating cryptocurrencies into global financial markets introduces risks and opportunities for investors and policymakers. The findings underscore the need for market participants to account for these volatility interactions in their risk management strategies. Additionally, policymakers must consider these interlinkages to maintain financial stability. This research contributes to the literature on financial market volatility by emphasising the importance of understanding the impact of emerging digital currencies on traditional stock markets.

Article History

Received:
02-Jul-2024

Revised:
10-Aug-2024

Re-revised:
03-Sep-2024

Accepted:
04-Sep-2024

Published:
17-Sep-2024

Keywords: Cryptocurrency, Pakistan Stock Exchange, Stock market, Volatility spillover, ADF Test, DCC GARCH Model, MGARCH, BEKK model, PSX.

How to Cite: Hussain, I., Ali, N., Ahmad, H. B., Ashraf, S. (2024). Volatility spillover effect between cryptocurrency and stock market using MGARCH BEKK model. *Natural and Applied Sciences International Journal (NASIJ), 5*(2), 32-55. https://doi.org/10.47264/idea.nasij/5.2.3

1. Introduction

The high-speed development of financial markets has added cryptocurrencies as a vital asset class, unique from traditional ones due to their decentralized nature and more especially profiteering potential. It attracted a broad range of investors and raised questions regarding the relationship between cryptocurrencies and traditional stock markets. The benefits of understanding 'volatility spillovers' — that is, when volatility in one market has an effect on another— as part of risk management and financial security. This study uses the MGARCH BEKK model to analyse volatility spillovers between cryptocurrencies and stock markets, providing insights into portfolio diversification, investment strategies, and regulatory policies in the increasingly integrated global financial ecosystem.

A time series is stationary if its statistical properties, such as mean, variance, and autocorrelation, remain constant over time. In a stationary series, the relationship between values depends only on their time lag, not the actual time of observation. Many statistical models, like ARIMA, require stationarity. Non-stationary time series, on the other hand, exhibit changing statistical properties, such as shifting means and variances. To analyse these series, transformations like differencing and detrending are often necessary to achieve stationarity, ensuring more accurate predictions and analyses.

Traditional financial markets include equity (stock), bond and commodity trading. Stock exchanges like the NYSE and LSE make it easier for businesses to raise funds and for investors to get returns. The investors make a return on their investments by way of capital gains and dividends. Stock exchanges are in operation every working day and they are affected with the availability of news and the public's need for stocks.

- Order routing: Orders are sent from computer to computer
- Order execution: Also known as "click-and-trade"
- Credit risk management: Central counterparty trading
- Automated trade settlement: Also known as "straight-through processing"

Cryptocurrency markets: these are the online locations where people can trade various kinds of digital currencies like Bitcoin and Ethereum, using the cryptographic protocols.

Decentralized Cryptocurrencies: How they differ from traditional currency. Like we discussed, individuals have grown to love and accept cryptocurrencies for a variety of reasons.

Derivatives markets: In these markets, products are financial instruments that derive (i.e. their value depends on) the price of an underlying asset. The main types of derivatives traded in this type of market include futures, options and swaps. Derivatives are used for both hedging and speculation to transfer risk from those who cannot handle it (risk averters) to people that can manage the same perfectly well.

Financial markets volatility of the price change in financial instruments, determined using historical/implied volatility and realized. (Key Event) Factors: economic data, geopolitical event. The Pakistan Stock Market originated in 1947 with the introduction of Karachi Stock Exchange (KSE) just after independence, which was accompanied by only five listed companies.

Expand and growth: The market was growing, between the 1960s to the 1980s. It was established in 1970 and next to it met the needs of Islamabad Stock Exchange (ISE). Significant liberalization was introduced in the late 1990s, and a Securities and Exchange Commission of Pakistan (SECP) was established on April 30, 1999 to administer the designed market effects in Pakistan.

Current era: By 2012, the Karachi Stock Exchange (KSE) in addition to LSE and ISE merged by making PSX which improved system efficiency. Back in 2016, PSX sold off a stake of up to 40% in the exchange for cash investment and technical support from local as well as Chinese consortium.

The study aims to achieve the following objectives:

- To assess the Presence and Magnitude of Volatility Spillover Effects
- To investigate the Directionality of Volatility Spillovers
- To explore the Impact of Major Events on Volatility Spillovers

2. Literature review

This literature review introduces a flexible multivariate GARCH model that allows parameterization of large covariance matrices while maintaining ease of estimation. By incorporating unconditional information, the model reduces parameter estimation challenges and avoids convergence issues, demonstrated through both synthetic and real-world examples (Van der Weide, 2002).

The literature review explores extreme stock market volatility, particularly in the Indian market, driven by investor euphoria rather than fundamentals. It examines whether "New Economy" stocks contributed to a speculative bubble and investigates psychological factors behind volatility. Empirical analysis of representative stocks and the BSE Sensex over two years, alongside tests of regulatory measures like rolling settlement and dematerialization, aimed to enhance market efficiency and guide regulators and investors (Bandivadekar & Ghosh, 2003).

Batra explores changes in Indian stock market volatility from 1979 to 2003, focusing on the impact of financial liberalization. Using monthly returns and the asymmetric GARCH model, it examines volatility persistence and market cycles. The study finds increased volatility post-BOP crisis and reforms, driven more by domestic factors than global events, with overall lower volatility and more stable market cycles in the post-liberalization era (Batra, 2004). Described due to the insufficient integration, it appears that adding mainland Chinese companies to an investment portfolio would have helped foreign investors reduce total portfolio risk by reducing diversifiable risk (Lanne & Saikkonen, 2007).

This study examines multivariate GARCH models in vector form, highlighting the limitations of expressing vectorized positive semidefinite matrices in the basic BEKK model. Through linear algebraic analysis, it demonstrates that these matrices can be projected into a strict subset of themselves. Additionally, the study provides a linear algebraic result, showing model similarities in the second dimension and presenting a clear, analytically feasible example of a VEC model without a BEKK representation in three dimensions (Stelzer, 2008).

The day of the week effect study concentrated on an oddity in Pakistan's equity market practices related to stocks. The daily stock prices associated with the KSE-100 Index from January 2006 to December 2010 were the modus operandi used in this study. Five days made up the working week for trading matters. According to the study, Tuesday returns were both favourable and very significant. Thus, it was concluded that the Pakistani stock market experienced a day effect. Compared to the other days, Tuesday's returns were higher on average. The purpose of this study was fulfilled by performing regression analysis (Hussain *et al.*, 2011).

This research examines working capital management strategies and productivity in Pakistan's manufacturing sector, analysing data from 37 companies (2009-2014). Using regression analysis and Data Envelopment Analysis (DEA), findings reveal that 15 firms need increased inputs for higher output, while 6 require input reductions. Tobit regression shows leverage improves efficiency, while collection duration decreases it (Ahmad *et al.*, 2017).

This study evaluates the weak-form market efficiency of stock market returns in 12 Asian-Pacific countries from January 2004 to December 2009. Using tests like autocorrelation and variance ratio, it concludes that not all countries' stock prices follow random walks, offering arbitrage opportunities (Hamid *et al.*, 2017).

Observed that cryptocurrencies recently emerged as a popular asset class, attracting investors with a high-risk appetite and speculative tendencies. They were not backed by physical assets, such as commodities or real currencies; instead, they were purely speculative assets characterized by high volatility. Regulatory authorities worldwide had conflicting rules regarding cryptocurrencies. Recent studies on cryptocurrency volatility primarily focused on univariate volatility analysis and volatility spillover between cryptocurrencies and other asset classes, mostly stocks and commodities (Bhattacharya *et al.*, 2022).

The study examined how COVID-19 heightened global financial market volatility and analysed risk transfer between the cryptocurrency market and global stock indices. Using the Constant Conditional Correlation Multivariate GARCH model on daily prices from December 2019 to July 2020, the analysis revealed significant volatility transmission between Bitcoin and major stock indices. These findings aim to aid investors in making informed portfolio decisions (Atici Ustalar *et al.*, 2022).

This study modelled the volatility of Bitcoin, Dash, Monero, and Stellar, identified structural breaks, and explored their connections with US equity, bond markets, and COVID-19 impacts. Using a comparative GARCH model, ICSS algorithm, and SEM, it found return-volatility spillovers among Bitcoin, Dash, and Stellar, with Monero as the main shock transmitter. No link with the US bond market was found, but cryptocurrency prices were affected by US energy market issues and pandemic-related uncertainty. The findings stress the need for timely risk management interventions (Bouteska *et al.*, 2023).

This study examined volatility spillover across four major exchanges and six liquid cryptocurrencies using high-frequency data. Results reveal that Ripple attracted more funds on Coinbase but contributed more on other exchanges. Bitfinex and Binance exhibited distinct net spillover impacts on the cryptocurrency markets. The analysis highlights the dynamic heterogeneity of exchanges in terms of volatility spillover, identifying key factors influencing overall market connectivity, particularly in the interconnection between cryptocurrencies and exchanges (Wu *et al.*, 2024). Mishra and Dash (2024) proposed that this study aimed to investigate the conditional volatility of the Asian stock markets in relation to Bitcoin and global crude oil price movements.

Rastogi and Kanoujiya (2024) explained that the main aim of the study was to explore the volatility spillover effect of cryptocurrencies (Bitcoin, Ethereum, and Litecoin) on inflation volatility in India. The study utilized the Bivariate GARCH model (BEKK-GARCH), a widely used tool for analysing volatility spillover effects. Monthly data on cryptocurrencies and inflation (WPI and CPI indices) were collected from 2015 to 2021.

3. Methodology

The methodology is designed to rigorously examine the dynamic interrelationship between these two markets using advanced econometric and statistical techniques. It introduces the time series analysis and basic notions like stationarity, volatility and spillover effect. It also includes tests for checking stationarity, e.g. graphical analysis, unit root test (ADF – test). The study defines the statistical techniques to convert non- stationarity series into stationarity, hypothesis and data cleaning process. The study also introduces the multivariate Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models used in this research. MGARCH models are considered as one of the most useful and popular tools for analysing and modelling time varying volatility and spillover effect of multiple time series. The study explains the volatility models and their parameters like CCC, DCC, and BEKK model.

Time series analysis examines data recorded at regular intervals to understand trends, patterns, and variations over time. It helps in decision-making, accurate predictions, and understanding historical behaviour. This technique is vital in fields like finance, economics, meteorology, and engineering, leveraging statistical models and machine learning for forecasting future values based on historical data.

The study adopts a quantitative research design, utilizing time series econometric between the cryptocurrency market, represented by Bitcoin (BTC), and the Pakistan stock market, represented by the KSE-100 index. The research aims to identify and quantify the extent to which volatility in the cryptocurrency market affects the volatility in the Pakistan stock market and vice versa.

The primary hypotheses of this study are:

- H1: There is a significant volatility spillover from the cryptocurrency market to the Pakistan stock market.
- H2: There is a significant volatility spillover from the Pakistan stock market to the cryptocurrency market.

For performing this study, we have collected the Pakistan stock market indexes closing prices as well also Cryptocurrency closing price all over time for a specific period of time on daily basis. Investigation has gone through seven cryptocurrencies: XRP, USDT, TRX DOGE Bitcoin (BTC) Binance Coin (BNB), and momentum patrol. KSE-100: The index measures the performance of largest companies by market capitalization listed on KARACHI Stock Exchange. The data spans a large time-byte of market conditions from January 01, 2019, to

April 15, 2024 (different economic cycles, bull and bear markets & major financial events). Cryptocurrency closing values for each day were acquired from reputable exchanges like Coin Market Cap and Coin desk. and Yahoo Finance.

The KSE-100 index's daily closing values were obtained either directly from the historical data of the Karachi Stock Exchange or from financial databases like investing.com or from Pakistan Stock Exchange (PSX) and Yahoo Finance. Data cleaning involves removing missing values and outliers. Transformation includes computing log returns. Stationarity is tested using the Augmented Dickey-Fuller (ADF) test. Missing data is addressed with interpolation or forward/backward filling. Outliers are detected through statistical tests and visual methods, then minorized or replaced with median values to ensure accuracy. To model and analyze the volatility dynamics, the study employs (GARCH) models. The GARCH model, introduced by Bollerslev (1986).

3.1. Multivariate GARCH models

The BEKK model is renowned for its capacity to capture asymmetric effects, such as the difference in impact between positive and negative shocks, in the volatility of asset returns.

The multivariate GARCH model are defined as:

$$r_t = \mu_t + \epsilon_t \tag{1}$$

$$\varepsilon_t = H_{t1/2} Z_t \tag{2}$$

Notations:

r_t : N×1 vector of log returns of N assets at time t.

ϵ_t : N×1 vector of mean-corrected returns of N assets at time t, i.e. $[\epsilon_t]$ = 0.

Cov $[\epsilon_t]$= H_t . μ_t : N×1 vector of the expected value of the conditional r_t .

H_t : N×N matrix of conditional variances of ϵ_t at time t.

$H_{t1/2}$: Any N×N matrix at time t

Z_t: N×1 vector of IID errors such that E (Z_t) = 0 and $E\ [Z_t Z_t']$ = IN where IN is an N-dimensional identity matrix. The conditional variance matrix of the multivariate process is given by: The conditional variance matrix of the multivariate process is given by: Var (r_t)=Var (ϵ_t) = $H_{t1/2}$Var $(Z_t)H\text{`}\ t$ 1/2 =H

3.2. DCC-GARCH model

The Dynamic Correlation GARCH (DCC) model of Engle (2002) is a different multivariate approach, in which the correlation between multiple time series can vary with time. By doing so, the DCC model acknowledges time-varying correlations in the analysis: this not only improves the risk assessment, portfolio optimization and asset allocation strategies of finance but also arms investors with reliable techniques when it comes to managing market-related risks.

r_t: N×1 vector of log returns of N assets at time t.

ϵ_t: N×1 vector of mean-corrected returns of N assets at time t, i.e., E [ϵ_t] = 0.
Cov [ϵ_t] = H_t.

μ_t: N×1 vector of the expected value of the conditional r_t.

H_t: N×N matrix of conditional variances of ϵ_t at time t.

H_t^ (1/2): Any N×N matrix at time t such that H_t is the conditional variance matrix of ϵ_t. H_tt^ (1/2) may be obtained by a Cholesky factorization of H_t.

D_t: N×N diagonal matrix of conditional standard deviations of ϵ_t at time.

R_t: N × N Conditional correlation matrix of ϵ_t at time t.

Z_t: N×1 vector of IID errors such that E (Z_t)=0 and E [$Z_t Z_t{}'$] = I

3.3. BEKK model

The BEKK (Baba, Engle, Kraft, and Kroner) specification is a multivariate GARCH model that aims to accurately represent and calculate the changing covariance between various time series. The BEKK model, proposed by Engle and Kroner in 1995, overcomes several drawbacks of previous multivariate GARCH models by guaranteeing that the conditional covariance matrix is positive definite.

The BEKK model is defined in the following manner:

$$H(t) = \mathrm{C}'\mathrm{C} + \mathrm{A}'(\varepsilon_{t-1})(\varepsilon_{t-1})' + \mathrm{B}'H_{t-1}B \tag{3}$$

$$H_t = \mathrm{C}' * \mathrm{C} + \mathrm{A}' * (\varepsilon_{t-1})(\varepsilon_{t-1})' + \mathrm{A} * \mathrm{B}' * {}'H_{t-1}B \tag{4}$$

The equation can be expressed as H(t) is the conditional covariance matrix at time t. C' and C is a matrix that has zeros above the main diagonal. Matrix A and matrix B are both parameter matrices. The symbol ϵ represents the vector of error terms, denoted as t or ϵ.

The Maximum Likelihood Estimation (MLE) technique used estimate model parameters. Under the assumption of normally distributed errors, MLE guarantees impartial and effective estimations. To get the estimations, the log-likelihood function was maximized.

To make sure the model was adequate, post-estimation diagnostic tests were carried out. Moreover, to see if there is remaining ARCH effects in standardized residuals, use Engle's ARCH Test, Ljung-Box Experiment was used to check the residuals and squared residuals for autocorrelation.

Portmanteau multivariate test to determine whether the residuals have cross-correlation. Stationarity and diagnostic tests are vital in time series analysis. Stationarity, tested using Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests, ensures stability of statistical parameters over time. These tests confirm that the log return series is stationary, a prerequisite for reliable modelling.

Diagnostic tests, including the Ljung-Box Test for autocorrelation, ARCH-LM Test for ARCH effects, and Q-Q Plot for normality of residuals, assess model adequacy and reliability. Together, these tests ensure robust and accurate econometric analysis.

Forecast Error Variance Decomposition (FEVD) is used to quantify the proportion of the forecast error variance of each variable that is attributable to shocks to each variable of VAR model. It decomposes the variance of the forecast errors into components attributable to each variable. FEVD helps identify the relative importance of shocks to cryptocurrency KSE in explaining the variability of each market.

3.4. Software and tools

The analysis utilizes R (with "rugarch", "vars", "tseries" packages) for data preprocessing and econometric modelling, Python ("pandas", "numpy", "matplotlib", "statsmodels") for data manipulation and visualization, and EViews for time series analysis and advanced econometric modelling.

4. Analysis and results

4.1. Descriptive statistic

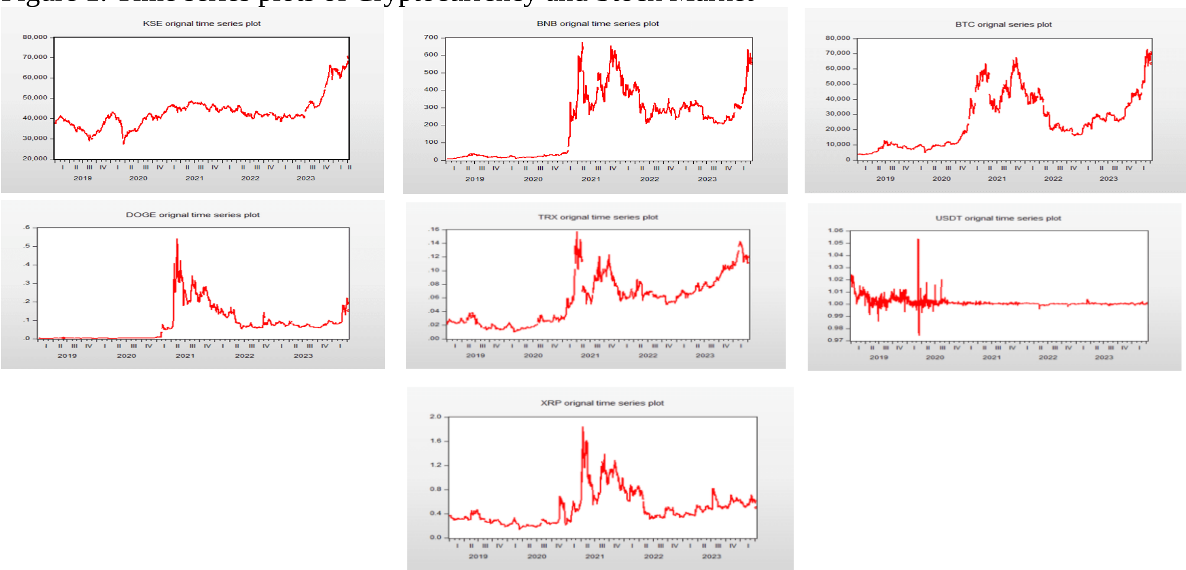
Table-1 presents the descriptive statistics for the Karachi Stock Exchange (KSE) and six cryptocurrencies: Bitcoin (BTC), Dogecoin (DOGE), XRP, Tether (USDT), TRON (TRX), and Binance Coin (BNB). The mean values indicate the average level of each series during the sample period. KSE has a mean value of 42875.21, while BTC has an average of 23062.87, which is significantly higher than the other cryptocurrencies. DOGE and TRX have the lowest mean values at 0.064447 and 0.054139, respectively.

Table-1: Descriptive statistic

Statistic	KSE	BTC	DOGE	XRP	USDT	TRX	BNB
Mean	42875.21	23062.87	0.064447	0.524238	1.001284	0.054139	177.2603
Median	41955.56	18342.63	0.014233	0.448453	1.000348	0.054444	44.40133
Maximum	70544.58	73083.50	0.541334	3.196630	1.077880	0.199655	675.6841
Minimum	27228.80	3242.485	0.001313	0.139635	0.966644	0.002062	1.510360
Std. Dev.	6981.559	17170.08	0.085314	0.325503	0.005312	0.033235	178.2139
Skewness	1.405386	0.835874	1.847401	2.494792	2.791951	0.622443	0.627239
Kurtosis	6.248270	2.665860	6.793109	14.23788	43.86080	2.690467	2.231543
Jarque-Bera	1216.274	191.5796	1848.253	9965.677	112110.1	108.4692	142.6597

The median values reveal the midpoint of each series: KSE's median is 41,955.56, slightly lower than its mean; BTC's median is 18,342.63, also below its mean; DOGE's median is 0.014233, while TRX's median is 0.054444, close to its mean. KSE ranges from 27,228.80 to 70,544.58, while BTC ranges widely from 3,242.485 to 73,083.50. DOGE ranges from 0.001313 to 0.541334, and TRX from 0.002062 to 0.199655. The series shows high volatility, positive skewness, and peaked distributions, with all failing the Jarque-Bera test for normality.

Figure 1: Time series plots of Cryptocurrency and Stock Market



The time series plots in Figure 1 from January 1, 2019, to May 14, 2024, reveal significant insights into the price movements of various assets. The Karachi Stock Exchange (KSE) shows substantial fluctuations, beginning around 41,000 points, peaking above 70,000, and dropping to 27,000 before stabilizing near 42,000, reflecting a complete market cycle influenced by economic events and investor sentiment. Bitcoin (BTC) displays notable volatility, with a significant upward trend and rapid price swings, ultimately stabilizing at a higher level, demonstrating its growing market prominence.

4.2. Stationarity test

Stationarity tests are normally done in time series analysis to determine if a dataset is stable over the period, hence the essentials of stationarity tests like Phillips-Perron (PP) or Augmented Dickey-Fuller (ADF). Stationarity is a crucial feature of many statistical modelling approaches such as autoregressive models and various types of GARCH model. If data is non-stationary, models can produce invalid and false predictions due to the changing statistical properties (i.e. mean, variance, autocorrelation) over time impacting model interpretation. For unit root testing, the PP and ADF tests are very common methods to check if a time series is stationary or not, which affects whether relies built on that data make any sense. Whereas these tests confirm the stationarity of a time series, and which leads to have more reliable forecasting, well model fit and powerful inference, hence can draw smarter conclusions based on this data from such a valuable source.

Presented in Table-2 are the results for Karachi Stock Exchange (KSE) and six cryptocurrencies: Binance Coin (BNB), Bitcoin, Dogecoin, TRON; Tether, XRP from Phillips-

Perron Test significant at first difference. Three times, first with a constant term and then for both the case without present of trend effects as well as stationarity in all series were performed: using three models (with constant, constant, and no) w.r.t. significant t-statistics along with p-. values = 0.000 set at each level/mode respectively. This suggests that each series is stationary and useable for further time-series analysis.

Table-2: Unit Root Test (PP) at first difference

		KSE	BNB	BTC	DOGE	TRX	USDT	XRP
With Constant	t-Statistic	-31.810	-38.899	-36.574	-34.465	-40.091	-135.085	-36.859
	Prob.	0.000	0.000	0.000	0.000	0.000	0.000	0.000
With Constant and Trend	t-Statistic	-31.820	-38.890	-36.568	-34.450	-40.081	-137.758	-36.845
	Prob.	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Without Constant and Trend	t-Statistic	-31.786	-38.882	-36.547	-34.474	-40.106	-134.048	-36.873
	Prob.	0.000	0.000	0.000	0.000	0.000	0.000	0.000

4.3. CCC-GARCH model (1)

Table-3 provides the results of CCC-GARCH (1, imp) model for KSE, BTC, BNB and USDT. Among others, KSE is characterized by quite significant volatility persistence (Alpha1 = 0.13854**) and GARCH-like dynamics (Beta1 = 0.80655**). BTC and BNB also have very clear volatility persistence, with BTC a high constant variance term having increasing returns-arch effects (Alpha1 = 0.30528), and similar ARCH-GARCH terms in Beta1 & Alpha2 for both BNB (Beta = 3.15149/0). USDT has significant GARCH term while slightly less with its ARCH term Significant correlations among BTC and BNB (rho_32 = 0.70014), but similar to KSE, there are none between the other two lines cryptocurrencies as well; proposing low direct effect of cryptocurrency on KSE which is shown below in the major diversification advantages by Cohen's coefficients.

Table-3: CCC-GARCH (1,1) model (1) results

Variables	Coefficients	Std Error	T-Stat	p-value
Part: KSE				
Cst(M)	0.00071	0.00029	2.48500	0.01310
Cst(V) x 10^4	0.07221	0.02486	2.90500	0.00370
ARCH(Alpha1)	0.13854	0.03076	4.50400	0.00000
GARCH(Beta1)	0.80655	0.03933	20.51000	0.00000
Part: BTC				
Cst(M)	0.00001	0.00117	0.00549	0.99560
Cst(V) x 10^4	2.26599	0.96343	2.35200	0.01880
ARCH(Alpha1)	0.14078	0.07008	2.00900	0.04480
GARCH(Beta1)	0.75404	0.08131	9.27300	0.00000

Variables	Coefficients	Std Error	T-Stat	p-value
Part: BNB				
Cst(M)	-0.00075	0.00122	-0.61470	0.53880
Cst(V) x 10^4	1.35013	0.76242	1.77100	0.07680
ARCH(Alpha1)	0.15290	0.04577	3.34100	0.00090
GARCH(Beta1)	0.81598	0.05952	13.71000	0.00000
Part: USDT				
Cst(M)	0.00006	0.00004	1.36100	0.17380
Cst(V) x 10^6	0.04213	0.01908	2.20900	0.02740
ARCH(Alpha1)	1.20422	0.83313	1.44500	0.14860
GARCH(Beta1)	0.41434	0.17609	2.35300	0.01880
Part: Correlation				
rho_21	0.00341	0.03783	0.09020	0.92810
rho_31	-0.00996	0.03732	-0.26680	0.78970
rho_41	-0.01498	0.03583	-0.41800	0.67600
rho_32	0.70014	0.02414	29.01000	0.00000
rho_42	-0.03054	0.07924	-0.38540	0.70000
rho_43	-0.02816	0.05957	-0.47280	0.63640

4.4. CCC-GARCH model (2)

The second stock Dogecoin (DOGE) and TRON (TRX), XRP on the Karachi Stock Exchange composite index are collectively evaluated using CCC-GARCH (1,1). Volatility persistence and the presence of other asymmetric ARCH-GARCH effects at different autoregressive lags is clear from Table-4, where high vol path continues to KSE (also DOGE N TRX in this set), all strong ARCH GARCH effect. The KSE is characterized by high volatility; however, the cryptocurrencies do not reveal a negative relationship between them as there are no significant correlations with DOGE, TRX or XRP. In sharp contrast, DOGE and XRP show complex fragility with a high level of correlation. Such insights are vital for effective risk management and informed investment decision-making in highly volatile markets.

The CCC-GARCH (1,1) model shows a strong correlation between the Karachi Stock Exchange and cryptocurrencies, with DOGE, TRX, and XRP showing a medium to high correlation, indicating significant collaboration.

Table-4: CCC-GARCH (1,1) model (2) results

Variables	Coefficients	Std Error	T-Stat	p-value
Part: KSE				
Cst(M)	0.00072	0.00028	2.55600	0.01070
Cst(V) x 10^4	0.07355	0.02512	2.92800	0.00350
ARCH(Alpha1)	0.14153	0.03129	4.52400	0.00000

Variables	Coefficients	Std Error	T-Stat	p-value
GARCH(Beta1)	0.80289	0.03978	20.18000	0.00000
Part: DOGE				
Cst(M)	-0.00155	0.00163	-0.94790	0.34330
Cst(V) x 10^4	0.64063	0.46034	1.39200	0.16430
ARCH(Alpha1)	0.15304	0.05580	2.74300	0.00620
GARCH(Beta1)	0.88573	0.03532	25.07000	0.00000
Part: TRX				
Cst(M)	0.00155	0.00093	1.67300	0.09450
Cst(V) x 10^4	0.32768	0.19278	1.70000	0.08940
ARCH(Alpha1)	0.15256	0.04912	3.10600	0.00190
GARCH(Beta1)	0.86134	0.03665	23.50000	0.00000
Part: XRP				
Cst(M)	-0.00233	0.00185	-1.26100	0.20760
Cst(V) x 10^4	10.35066	6.40590	1.61600	0.10640
ARCH(Alpha1)	0.40499	0.31215	1.29700	0.19470
GARCH(Beta1)	0.45595	0.25358	1.79800	0.07240
Part: Correlation				
rho_21	-0.00761	0.02764	-0.27550	0.78300
rho_31	-0.04664	0.03117	-1.49600	0.13480
rho_41	-0.01076	0.03164	-0.34020	0.73380
rho_32	0.02290	0.03636	0.62980	0.52890
rho_42	0.20007	0.04183	4.78300	0.00000
rho_43	-0.00239	0.03054	-0.07814	0.93770

4.5. DCC-GARCH model (1)

The DCC-GARCH (1,1) Model (1) analyses dynamic conditional correlations and volatilities among the Karachi Stock Exchange (KSE), Bitcoin (BTC), Binance Coin (BNB), and Tether (USDT). Table 5 reveals significant volatility persistence for all assets, with notable ARCH and GARCH effects. The dynamic correlations show minimal direct impact of cryptocurrencies on KSE, with significant interdependence among cryptocurrencies, especially between BTC and BNB. This model highlights the complex interactions and provides insights into co-movement and diversification benefits, aiding portfolio management and risk assessment in volatile markets.

Table-5: DCC-GARCH (1,1) model (1) results

Variables	Coefficients	Std Error	T-Stat	p-value
Part: KSE				
Cst(M)	0.00072	0.00029	2.49400	0.01270

Variables	Coefficients	Std Error	T-Stat	p-value
Cst(V) x 10^4	0.06892	0.02377	2.90000	0.00380
ARCH(Alpha1)	0.14295	0.03029	4.72000	0.00000
GARCH(Beta1)	0.81059	0.03682	22.01000	0.00000
Part: BTC				
Cst(M)	0.00125	0.00118	1.05600	0.29100
Cst(V) x 10^4	1.79361	1.06600	1.68300	0.09270
ARCH(Alpha1)	0.13421	0.08308	1.61500	0.10650
GARCH(Beta1)	0.78926	0.10429	7.56800	0.00000
Part: BNB				
Cst(M)	0.00092	0.00123	0.74390	0.45710
Cst(V) x 10^4	1.30975	0.72587	1.80400	0.07140
ARCH(Alpha1)	0.13849	0.04294	3.22500	0.00130
GARCH(Beta1)	0.82810	0.05619	14.74000	0.00000
Part: USDT				
Cst(M)	0.00006	0.00005	1.29500	0.19550
Cst(V) x 10^6	0.03909	0.01856	2.10700	0.03530
ARCH(Alpha1)	1.13566	0.88521	1.28300	0.19970
GARCH(Beta1)	0.44912	0.20224	2.22100	0.02650
Part: Correlation				
rho_21	0.02464	0.05734	0.42980	0.66740
rho_31	0.00144	0.05772	0.02501	0.98010
rho_41	-0.05214	0.05318	-0.98040	0.32710
rho_32	0.73420	0.03341	21.98000	0.00000
rho_42	-0.05317	0.10794	-0.49260	0.62240
rho_43	-0.00490	0.07716	-0.06350	0.94940
Alpha	0.02144	0.00576	3.72100	0.00020
Beta	0.94998	0.01514	62.75000	0.00000

4.6. DCC-GARCH model (2)

The DCC-GARCH (1,1) model analyses the correlation and volatility of Karachi Stock Exchange (KSE), Dogecoin (DOGE), TRON (TRX), and XRP, capturing the time difference between positive and weak spillover effects. It helps investors and policymakers manage risk and optimize information in volatile markets.

DCC-GARCH (1,1) model showing the time-varying conditional correlations between KSE and respective crypto-currencies DOGE-TRX-XRP; for each DCC equation ≠ 0 at any confidence level. The relationship between KSE and the cryptocurrencies is weak with negative or close to zero smoothed correlation. More importantly, KSE is paired with DOGE around -0.01 to 0.01 correlations while both TRX and XRP have similar weak relationships against it This indicates that KSE works autonomously from these digital resources.

Table-6: DCC-GARCH (1,1) model results for model (2)

Variables	Coefficients	Std Error	T-Stat	p-value
Part: KSE				
Cst(M)	0.00073	0.00028	2.58500	0.00990
Cst(V) x 10^4	0.07385	0.02523	2.92700	0.00350
ARCH(Alpha1)	0.14147	0.03128	4.52300	0.00000
GARCH(Beta1)	0.80254	0.03994	20.09000	0.00000
Part: DOGE				
Cst(M)	-0.00161	0.00162	-0.98990	0.32240
Cst(V) x 10^4	0.64470	0.46192	1.39600	0.16300
ARCH(Alpha1)	0.15328	0.05586	2.74400	0.00620
GARCH(Beta1)	0.88558	0.03533	25.07000	0.00000
Part: TRX				
Cst(M)	0.00158	0.00093	1.69600	0.09010
Cst(V) x 10^4	0.32791	0.19302	1.69900	0.08960
ARCH(Alpha1)	0.15403	0.04945	3.11500	0.00190
GARCH(Beta1)	0.86053	0.03667	23.47000	0.00000
Part: XRP				
Cst(M)	-0.00231	0.00187	-1.24100	0.21500
Cst(V) x 10^4	10.40761	6.47000	1.60900	0.10800
ARCH(Alpha1)	0.40433	0.31131	1.29900	0.19420
GARCH(Beta1)	0.45437	0.25474	1.78400	0.07470
Part: Correlation				
rho_21	-0.00690	0.02774	-0.24890	0.80350
rho_31	-0.04691	0.03144	-1.49200	0.13600
rho_41	-0.01130	0.03149	-0.35890	0.71970
rho_32	0.02660	0.03709	0.71720	0.47340
rho_42	0.20292	0.04269	4.75400	0.00000
rho_43	-0.00193	0.03087	-0.06246	0.95020
Alpha	0.00747	0.01158	0.64540	0.51880
Beta	0.31251	0.45141	0.69230	0.48890

4.7. BEKK-GARCH Model (1)

The BEKK-GARCH (1,1) model (1) shows the relationship between the Karachi Stock Exchange (KSE), Bitcoin (BTC), Binance Coin (BNB), and Tether (USDT). The relationship between KSE and cryptocurrencies is generally low, ranging from zero, indicating a direct relationship. For example, the relationship between KSE and BTC oscillates between -0.05 and 0.05, similar to the relationship between KSE and BNB and USDT.

Table 7: BEKK-GARCH (1,1) model (1) results

Variables	Coefficients	Std Error	T-Stat	p-value
Cst1	0.00066	0.00027	2.40900	0.01610
Cst2	0.00207	0.00102	2.02900	0.04270
Cst3	0.00181	0.00115	1.57700	0.11510
Cst4	0.00001	0.00002	0.66310	0.50740
C_11	0.00300	0.00057	5.25100	0.00000
C_12	0.00040	0.00060	0.65990	0.50940
C_13	0.00012	0.00062	0.20010	0.84150
C_14	-0.00001	0.00001	-1.17600	0.23970
C_22	0.00734	0.00193	3.80200	0.00020
C_23	0.00568	0.00110	5.17700	0.00000
C_24	-0.00000	0.00002	-0.20640	0.83650
C_33	0.00519	0.00107	4.84500	0.00000
C_34	0.00002	0.00003	0.82650	0.40870
C_44	0.00013	0.00001	13.23000	0.00000
b_1.11	0.91860	0.02474	37.13000	0.00000
b_1.22	0.96481	0.01382	69.80000	0.00000
b_1.33	0.96334	0.00787	122.40000	0.00000
b_1.44	0.90924	0.02702	33.65000	0.00000
a_1.11	0.29876	0.05109	5.84800	0.00000
a_1.22	0.21223	0.04672	4.54300	0.00000
a_1.33	0.22912	0.02834	8.08500	0.00000
a_1.44	0.41627	0.09862	4.22100	0.00000

4.8. BEKK-GARCH model (2)

The conditional correlations for the KSE, DOGE, TRX and XRP by BEKK-GARCH (1, 1) are also shown in Table 8. The correlations with KSE and the cryptocurrencies are very, very low on average hovering around 0%, implying there is little to no direct impact. Conversely, correlations are somewhere near -0.1 and 0.1 when KSE is paired with DOGE just like those of TRX or XRP to it — all this further rests my case.

Table-8: BEKK-GARCH (1,1) model (2) results

Variables	Coefficients	Std Error	T-Stat	p-value
Cst1	0.00081	0.00030	2.73500	0.00630
Cst2	-0.00025	0.00305	-0.08244	0.93430
Cst3	0.00148	0.00116	1.27800	0.20150
Cst4	0.00036	0.00190	0.18940	0.84980

Variables	Coefficients	Std Error	T-Stat	p-value
C_11	0.00256	0.00041	6.16400	0.00000
C_12	-0.00574	0.00678	-0.84640	0.39750
C_13	-0.00012	0.00042	-0.28560	0.77530
C_14	-0.00203	0.00289	-0.70040	0.48380
C_22	0.04870	0.00832	5.85000	0.00000
C_23	0.00044	0.00164	0.26860	0.78830
C_24	0.01680	0.01183	1.42000	0.15580
C_33	0.00195	0.00197	0.98760	0.32350
C_34	-0.00005	0.00025	-0.19970	0.84170
C_44	0.00006	0.00009	0.69680	0.48600
b_1.11	0.93554	0.01396	67.00000	0.00000
b_1.22	0.23026	0.19471	1.18300	0.23720
b_1.33	0.98795	0.01012	97.67000	0.00000
b_1.44	-0.94170	0.07201	-13.08000	0.00000
a_1.11	0.27045	0.03585	7.54400	0.00000
a_1.22	0.97313	0.30933	3.14600	0.00170
a_1.33	0.15221	0.06588	2.31000	0.02100
a_1.44	0.20727	0.12130	1.70900	0.08770

4.9. Diagnostic test for model (1)

The diagnostics of the BEKK-GARCH (1,1) model assesses its ability to capture the volatility of the Karachi Stock Exchange, Bitcoin, Binance Coin and Tether. It involves analysing the variance of the residuals to ensure that relationships and changes over time are accurately represented, thus verifying the reliability of the model in terms of respecting management risk.

Table-9: Diagnostics test results of Squared Standardized Residuals for model (1)

Q-test	Series: KSE	Series: BTC	Series: BNB	Series: USDT
CCC-GARCH (1,1)				
Q (5)	8.82068 [0.116433]	1.69323 [0.889751]	3.07153 [0.688959]	0.557759 [0.989855]
Q (10)	14.0411 [0.171123]	3.84549 [0.954048]	5.71780 [0.838389]	1.03200 [0.999802]
Q (20)	31.5285 [0.148589]	8.23505 [0.990194]	11.4357 [0.934131]	2.62017 [0.999987]
Q (50)	67.6349 [0.248910]	27.0135 [0.996762]	58.5897 [0.189426]	4.84477 [1.000000]
DCC-GARCH (1,1)				
Q (5)	7.34995 [0.195887]	1.85714 [0.868537]	4.11709 [0.532684]	0.606742 [0.987695]
Q (10)	12.9368 [0.227225]	4.02136 [0.946378]	7.42732 [0.684580]	1.06579 [0.999766]
Q (20)	29.8105 [0.072983]	8.22718 [0.990259]	13.9344 [0.833804]	2.28068 [0.999996]
Q (50)	63.4114 [0.096383]	26.6924 [0.997216]	55.8380 [0.264801]	4.81757 [1.000000]

Q-test	Series: KSE	Series: BTC	Series: BNB	Series: USDT
BEKK-GARCH (1,1)				
Q (5)	20.1141 [0.051189]	1.51845 [0.910937]	28.2459 [0.067332]	2.29601 [0.806853]
Q (10)	25.2736 [0.064850]	2.47964 [0.991161]	33.1792 [0.084254]	2.65601 [0.988402]
Q (20)	39.4460 [0.085865]	5.03487 [0.999707]	40.0377 [0.103940]	3.14538 [0.999993]
Q (50)	71.2920 [0.125598]	14.7161 [0.999999]	64.4506 [0.132158]	7.02117 [1.000000]

For the Q-test, the results at different lags (5, 10, 20, and 50) for each series indicate the presence of autocorrelation in the squared residuals. For KSE, the Q (5) statistic is 8.82068 with a p-value of 0.116433, and Q (10) is 14.0411 with a p-value of 0.171123. These p-values suggest that there is no significant autocorrelation in the residuals at these lags. Similarly, BTC shows Q (5) of 1.69323 with a p-value of 0.889751 and Q (10) of 3.84549 with a p-value of 0.954048, indicating no significant autocorrelation. For BNB, Q (5) is 3.07153 with a p-value of 0.688959, and Q (10) is 5.71780 with a p-value of 0.838389, again suggesting no significant autocorrelation. USDT also shows no significant autocorrelation with Q (5) of 0.557759 and a p-value of 0.989855, and Q (10) of 1.03200 with a p-value of 0.999802.

Table-10: Diagnostics test results of Squared Standardized Residuals for model (1)

Hosking's test	Results_1	Li and McLeod's test	Results_2
CCC-GARCH (1,1)			
Hosking (5)	168.530 [0.094534]	Li-McLeod (5)	168.371 [0.075698]
Hosking (10)	224.005 [0.113433]	Li-McLeod (10)	223.986 [0.129876]
Hosking (20)	344.075 [0.150754]	Li-McLeod (20)	344.527 [0.146804]
Hosking (50)	812.208 [0.355612]	Li-McLeod (50)	813.644 [0.342469]
DCC-GARCH (1,1)			
Hosking (5)	184.146 [0.056037]	Li-McLeod (5)	183.956 [0.053914]
Hosking (10)	243.638 [0.085419]	Li-McLeod (10)	243.559 [0.093875]
Hosking (20)	363.551 [0.129828]	Li-McLeod (20)	363.940 [0.138632]
Hosking (50)	826.919 [0.232096]	Li-McLeod (50)	828.399 [0.221211]
BEKK-GARCH (1,1)			
Hosking (5)	280.184 [0.064324]	Li-McLeod (5)	279.776 [0.060743]
Hosking (10)	348.369 [0.076239]	Li-McLeod (10)	348.021 [0.135643]
Hosking (20)	457.210 [0.081738]	Li-McLeod (20)	457.470 [0.146405]
Hosking (50)	852.108 [0.115734]	Li-McLeod (50)	855.335 [0.217937]

4.10. Model selection

Table-11 presents the information criteria for model (1), which includes Akaike Information Criterion (AIC), Schwarz Bayesian Criteria (SBC), Shibata Index and Hannan-Quinn criterion. These are the criteria of Empirical results used for checking goodness-of-fit and complexity in

BEKK-GARCH (1,1) model implemented on The Karachi Stock Exchange (KSE), Bitcoin (BTC), Binance Coin (BNB and Tether (USDT).

AIC for Model (1): 0.01485 for this, we used the AIC to compare models: lower numbers mean that it fits better while taking care of model complexity. Model (1) appears to capture the best trade-off, according to AIC.

The Schwarz Information Criterion (SIC) for Model 1 was equal to 0.10186, in favour of correct models with the lowest complexity possible; Shibata criterion = 0.01430, which is at model (1) of the dynamic relationship capture (Table-11). The value of Hannan-Quinn Criterion (HQC) is 0.04749 indicating a balance between precision and simplicity The low values of AIC, SIC, Shibata and HQC all seem to suggest that BEKK-GARCH (1, 1) model is a good fit for the data capturing appropriate level of complexity in volatility dynamics.

Table-11: Information criteria for model (1)

Information Criteria	CCC-GARCH (1,1)	DCC-GARCH (1,1)	BEKK-GARCH (1,1)
Akaike	0.01485	0.01786	-24.37888
Schwarz	0.10186	0.11278	-24.29187
Shibata	0.01430	0.01720	-24.37943
Hannan-Quinn	0.04749	0.05346	-24.34625

4.11. Diagnostic test for model (2)

Well, the diagnostic test for model (2) under BEKK-GARCH (1,1) framework is one of those exercises good to do whenever we try and understand if our model truly captivated adequately the volatility dynamics among Karachi Stock Exchange (KSE), Dogecoin (DOGE), TRON (TRX), XRP. These tests are to know how well the model recaptures those time-varying correlations and volatilities within the data. Usually, to identify if any further autocorrelation or pattern remains and therefore the model is inadequate, we use diagnostic tests with standardized residuals and their squares. We follow the diagnosis test results in detail, and we can find whether these complicated interactions and volatility structures are RC as BELK-GARCH (1,1) model for model (2).

The Table-12 presents the diagnostic test results of squared standardized residuals for Model (2), which includes the Karachi Stock Exchange (KSE), Dogecoin (DOGE), TRON (TRX), and XRP. The diagnostics involve Q-tests, Hosking's tests, and Li and McLeod's tests, which assess whether the BEKK-GARCH (1,1) model effectively captures the volatility dynamics without leaving significant autocorrelation in the residuals.

For the Q-test, the results for KSE show Q (5) = 8.50957 (p = 0.130298), Q(10) = 13.6785 (p = 0.188170), Q(20) = 29.8009 (p = 0.073145), and Q(50) = 66.0815 (p = 0.063335). These p-values indicate that there is no significant autocorrelation in the squared standardized residuals at the 5%, 10%, and 20% lag levels, suggesting that the model adequately captures the volatility for KSE.

For DOGE, the Q-test results show Q (5) = 1.20925 (p = 0.943986), Q (10) = 1.85802 (p = 0.997317), Q (20) = 17.54745 (p = 0.056413), and Q (50) = 19.23727 (p = 0.068381). The high p-values indicate no significant autocorrelation at the 5%, 10%, and 50% lag levels, but the result for Q (20) is marginally significant, suggesting the model captures the volatility for DOGE adequately overall.

For TRX, the Q-test results show Q (5) = 0.09969 (p = 0.999838), Q (10) = 3.44123 (p = 0.969056), Q (20) = 10.53750 (p = 0.957346), and Q (50) = 34.10410 (p = 0.958162). These p-values are all very high, indicating no significant autocorrelation in the squared standardized residuals, confirming that the model captures the volatility dynamics for TRX effectively.

For XRP, the Q-test results show Q (5) = 0.710260 (p = 0.982400), Q(10) = 1.13953 (p = 0.999687), Q (20) = 1.84407 (p = 0.999999), and Q(50) = 9.10583 (p = 1.000000). The high p-values across all lag levels suggest no significant autocorrelation, indicating the model captures the volatility for XRP well.

Table-12: Diagnostics test results of Squared Standardized Residuals for model (2)

Q-test	Series: KSE	Series: BTC	Series: BNB	Series: USDT
CCC-GARCH (1,1)				
Q (5)	8.50957 [0.130298]	1.20925 [0.943986]	0.09969 [0.999838]	0.710260 [0.982400]
Q (10)	13.6785 [0.188170]	1.85802 [0.997317]	3.44123 [0.969056]	1.13953 [0.999687]
Q (20)	29.8009 [0.073145]	17.54745 [0.056413]	10.53750 [0.957346]	1.84407 [0.999999]
Q (50)	66.0815 [0.063335]	19.23727 [0.068381]	34.10410 [0.958162]	9.10583 [1.000000]
DCC-GARCH (1,1)				
Q (5)	8.54664 [0.128571]	1.21309 [0.943615]	0.103975 [0.999821]	0.71086 [0.982367]
Q (10)	13.6941 [0.187407]	1.87791 [0.997193]	3.47600 [0.967906]	1.13304 [0.999695]
Q (20)	29.9221 [0.071126]	176.603 [0.678734]	10.5597 [0.956852]	1.84997 [0.999999]
Q (50)	66.0487 [0.063674]	191.236 [0.736988]	34.1738 [0.957350]	9.13280 [1.000000]
BEKK-GARCH (1,1)				
Q (5)	25.7497 [0.075299]	4.34293 [0.501168]	15.1380 [0.091788]	1.87437 [0.866240]
Q (10)	33.7471 [0.095203]	4.58996 [0.916835]	15.8601 [0.103706]	2.17652 [0.994799]
Q (20)	52.0038 [0.125113]	239.101 [0.452782]	20.3669 [0.435196]	3.33896 [0.999989]
Q (50)	80.9175 [0.143675]	251.575 [0.471737]	36.4105 [0.924765]	36.9820 [0.914264]

Hosking's and Li and McLeod's tests corroborate the Q-test findings, with p-values indicating no significant autocorrelation in the squared standardized residuals for all series. For example, Hosking (5) for KSE is 47.8250 (p = 0.997192) and Li-McLeod (5) is 47.8992 (p = 0.997120), both indicating no significant residual autocorrelation. The diagnostic test results for Model (2) show that the BEKK-GARCH (1,1) model effectively captures the volatility dynamics for KSE, DOGE, TRX, and XRP. The lack of significant autocorrelation in the squared standardized residuals suggests that the model adequately models the conditional variances of these series. This confirms the robustness and adequacy of the BEKK-GARCH (1,1) model for analysing the volatility interactions among these financial assets, making it a reliable tool for risk management and investment decision-making.

Table-13: Diagnostics test results of Squared Standardized Residuals for model (2)

Hosking's test	Results_1	Li and McLeod's test	Results_2
CCC-GARCH (1,1)			
Hosking (5)	47.8250 [0.997192]	Li-McLeod (5)	47.8992 [0.997120]
Hosking (10)	27.8411 [0.563782]	Li-McLeod (10)	27.8981 [0.563823]
Hosking (20)	55.9912 [0.782376]	Li-McLeod (20)	55.0354 [0.783616]
Hosking (50)	104.9781[0.732862]	Li-McLeod (50)	104.6548 [0.737194]
DCC-GARCH (1,1)			
Hosking (5)	47.8028 [0.997213]	Li-McLeod (5)	47.8770 [0.997141]
Hosking (10)	274.526 [0.764234]	Li-McLeod (10)	273.591 [0.763987]
Hosking (20)	553.708 [0.663789]	Li-McLeod (20)	550.759 [0.664325]
Hosking (50)	1048.71 [0.236523]	Li-McLeod (50)	1045.37 [0.236154]
BEKK-GARCH (1,1)			
Hosking (5)	96.7972 [0.073378]	Li-McLeod (5)	96.7576 [0.073761]
Hosking (10)	360.796 [0.147378]	Li-McLeod (10)	359.544 [0.147463]
Hosking (20)	712.544 [0.179345]	Li-McLeod (20)	708.144 [0.173692]
Hosking (50)	1510.40 [0.348965]	Li-McLeod (50)	1497.49 [0.348623]

4.12. Model selection

Table-14 presents the information criteria for model (2): AIC, SIC, Shibata Criterion and Hannan-Quinn Criterion. Such criteria are intended for measuring the suitability and complexity of BEKK-GARCH (1,1) model functioned on Pakistan Stock Exchange (PSX), Dogecoin ((DOGE), TRON (TRX) and XRP. Akaike Information Criterion: AIC of Model (2): 0.02203 The AIC is a model selection metric that describes the trade-off between how well our selected model fits with some data, while penalizing for increasing complexity. The smaller value of this method indicates better compromise between performance rating and simplicity. The AIC value puts model (2) in a favourable light as it is the best of both worlds which attests to its goodness of fit but not too complexity.

Table-14: Information Criteria for model (2)

Information Criteria	CCC-GARCH (1,1)	DCC-GARCH (1,1)	BEKK-GARCH (1,1)
Akaike	0.02203	0.02508	-14.86895
Schwarz	0.10904	0.12000	-14.78194
Shibata	0.02148	0.02443	-14.86951
Hannan-Quinn	0.05466	0.06069	-14.83632

The Shibata criterion for model (2) is 0.02148, similar to AIC. It is particularly useful for time series model selection, as lower values indicate a better model. Low Shibata criterion values support the suitability of model (2) to capture the relationship between KSE, DOGE, TRX, and XRP. HQC is another measure that balances model security and complexity, penalizing models lower than SIC but higher than AIC. HQC values indicate that model (2) achieves a good balance between capturing dynamic changes and controlling flexibility. The papers include KSE, DOGE, TRX, and XRP. The AIC, Shibata and HQC values show that the model captures the volatility dynamics and the interaction of financial series without too much complexity. However, the SIC values show that there is still room for simplification of the model. Overall, these results confirm the reliability of the model for further analysis and decision making on risk management and investment strategies related to these assets.

A variety of GARCH models are utilized to analyse co-movement and volatility spillovers between major cryptos including Bitcoin (BTC), Binance Coin (BNB) with the Karachi Stock Exchange (KSE). It was estimated using CCC-GARCH (1,1) and DCC GARCH (1,1) model reflecting time-varying correlation structure and a long memory characteristic for volatilities that are important in understanding market process. So, in order to better get data and apply the ARIMA model it was performed some statistics as plots of series with descriptive stats that showed high volatility (specially for BTC) and also time series plot which confirmed this need of starting differentiation at first. The results demonstrate notable internal associations among cryptocurrencies, mainly for BTC and BNB as well as their weak effect on KSE. This serves as a reminder for the need to investigate such interconnections when working on cryptocurrency risk management and investment strategies. The report provides useful lessons to investors and policymakers in both traditional and digital finance.

5. Conclusion

In this article, we investigate the influence of cryptocurrencies on traditional financial markets by examining volatility spillover effects. Cryptocurrencies have arisen as one of the defining asset classes this decade —they are both highly volatile and speculative in nature given limited regulatory frameworks. This combination has attracted a wide gamut of investors and created speculation on the interplay between cryptocurrencies & their links to traditional stock markets. The understanding of these dynamics is useful and valuable for effective risk management, portfolio diversification, or even as priorities to be aware in the prevention of systemic financial events. The study uses the BEKK model of Multivariate Generalized Autoregressive Conditional Heteroskedasticity (MGARCH) for examining volatility spillovers. Before getting to the study itself, a little background might be useful since we have discussed this research previously: The researchers' aim is both to understand what happens when cryptocurrencies—both well-regulated like bitcoin and anarchic like recently-the-Binance-hacked-remarkably-valuable Binance Coin—affect established markets and also (and more awkwardly) peculiar things they do. Studies of volatility spillovers based on the literature review are researched where they use different econometric models particularly MGARCH and highlight major results with methods adopted.

Acknowledgement:

We are thankful to the editor and reviewer for their valuable comments and suggestions. We are also thankful to our lab fellow and all those who assisted during this study.

Declaration of conflict of interest:

The author(s) declared no potential conflicts of interest(s) with respect to the research, authorship, and/or publication of this article.

Funding:

The author(s) received no financial support for the research, authorship and/or publication of this article.

ORCID iD:

Iqra Hussain https://orcid.org/0009-0000-1464-8447

Nazakat Ali https://orcid.org/0009-0005-9663-1935

Hafiz Bilal Ahmad https://orcid.org/0009-0003-1061-2174

Suhail Ashraf https://orcid.org/0009-0001-4302-6097

Publisher's Note:

IDEA PUBLISHERS (IDEA Publishers Group) stands neutral with regard to jurisdictional claims in the published maps and institutional affiliations.

References

Ahmad, M. F., Ishtiaq, M., Hamid, K., Khurram, M. U., & Nawaz, A. (2017). Data envelopment analysis and Tobit analysis for firm efficiency in perspective of working capital management in manufacturing sector of Pakistan. *International Journal of Economics and Financial Issues*, *7*(2), 706-713.

Atici Ustalar, S., Ayar, E., & Şanlisoy, S. (2022). The volatility transmission between cryptocurrency and global stock market indices: case of Covid-19 period. *İZMIR Journal of Economics*, *37*(2). 443-459. https://search.trdizin.gov.tr/tr/yayin/detay/1109983

Bandivadekar, S., & Ghosh, S. (2003). Derivatives and volatility on Indian stock markets. *Reserve Bank of India Occasional Papers, 24*(3), 187-201.

Batra, A. (2004). Stock return volatility patterns in India. *Indian Council for Research on International Economic Relations*.

Bhattacharya, R., Das, A., & Roy, S. (2022). Asymmetric volatility and volatility spillover: a study of major cryptocurrencies. *J. Commer. Account. Res*, *11*, 69-86.

Bouteska, A., Sharif, T., & Abedin, M. Z. (2023). Volatility spillovers and other dynamics between cryptocurrencies and the energy and bond markets. *The Quarterly Review of Economics and Finance*, *92*, 1-13. https://doi.org/10.1016/j.qref.2023.07.008

Hamid, K., Suleman, M. T., Ali Shah, S. Z., & Imdad Akash, R. S. (2017). Testing the weak form of efficient market hypothesis: Empirical evidence from Asia-Pacific markets. *Available at SSRN 2912908*.

Hussain, F., Hamid, K., Imdad Akash, R. S., & Imdad Khan, M. (2011). Day of the week effect and stock returns:(Evidence from Karachi stock exchange-Pakistan). *Far East Journal of Psychology and Business*, *3*(1), 25-31.

Lanne, M., & Saikkonen, P. (2007). A multivariate generalized orthogonal factor GARCH model. *Journal of Business & Economic Statistics*, *25*(1), 61-75. https://doi.org/10.1198/073500106000000404

Mishra, A., & Dash, A. K. (2024). Return volatility of Asian stock exchanges; a GARCH DCC analysis with reference of Bitcoin and global crude oil price movement. *Journal of Chinese Economic and Foreign Trade Studies*, 29-48, *17*(1) https://doi.org/10.1108/JCEFTS-01-2024-0009

Rastogi, S., & Kanoujiya, J. (2024). Impact of cryptos on the inflation volatility in India: an application of bivariate BEKK-GARCH models. *Journal of Economic and Administrative Sciences*, *40*(2), 221-237. https://doi.org/10.1108/JEAS-08-2021-0167

Stelzer, R. (2008). On the relation between the vec and BEKK multivariate GARCH models. *Econometric Theory*, *24*(4), 1131-1136.

Van der Weide, R. (2002). GO-GARCH: a multivariate generalized orthogonal GARCH model. *Journal of Applied Econometrics*, *17*(5), 549-564. https://doi.org/10.1002/jae.688

Wu, M., Wang, L., & Yang, H. (2024). Heterogeneity in the volatility spillover of cryptocurrencies and exchanges. *Financial Innovation*, *10*(1), 85. https://jfin-swufe.springeropen.com/articles/10.1186/s40854-023-00585-0