Natural and Applied Sciences International Journal (NASIJ)
eISSN: 2788-4619 (online)
https://www.ideapublishers.org/index.php/nasij
Vol. 5, No. 1 (January-June), 2024, pp. 168-181
https://doi.org/10.47264/idea.nasij/5.1.11
Research Article | Original Research

New memory-based ratio estimator in survey sampling

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Abstract:

This study proposes a new estimator based on the Exponential Weighted Moving Average (EWMA) statistic by following the concept of Noor-ul-Amin (2021). The EWMA model consumed contemporary and empirical data to enhance the competence of the population mean estimation. The memory type estimate is proposed with a twofold utilisation of Auxiliary Information (AUI) in alignment with the sampling type, i.e., Simple Random Sampling (SRS). A detailed numerical study and analysis are conducted to estimate the projected efficiency. This study provides an efficient estimator for population mean in the occurrence of time series data. The extra advantage of using EWMA statistics instead of classical statistics is that we can get better effectiveness of the mean estimate of the population under SRS by altering the significance of the smooth constant λ, as the cost of the smoothing constant λ decreases from one towards zero, and we gain the efficiency of the suggested estimator. For λ = 1, the proposed estimator performs the same as their comparative estimator. This expands incompetence over the presented ones statistically demonstrated in mean square error and relative effectiveness. Previous research is also accessed with everyday life information that indicates the conclusions of the simulation investigation.

Article History

Received:
12-Feb-2024

Revised:
28-Apr-2024

Re-revised:
10-May-2024

Accepted:
13-May-2024

Published:
28-June-2024

Keywords: Simple random sampling, Auxiliary information, Ratio estimator, Memory type estimate, Smoothing constant, Mean square, Relative efficiency, New estimator, Simulation.

How to Cite:
Aslam, I., Amin, M. N., Mahmood, A., & Sharma, P. (2024). New memory-based ratio estimator in survey sampling. *Natural and Applied Sciences International Journal (NASIJ)*, *5*(1), 168-181. https://doi.org/10.47264/idea.nasij/5.1.11

1. Introduction

The relevant estimation procedure supplies estimation with more precision. The Auxiliary Information (AUI) is integrated into these estimation procedures to obtain improved estimators. This auxiliary variable is also used at the beginning stages of designing or evaluation. Cochran (1940) suggests applying AUI at the estimate stage in the form of a ratio estimator, which is specified by Equation (1).

$$\bar{y}_r = \frac{\bar{y}}{\bar{x}}\bar{X} \tag{1}$$

Whereas the sample mean of the research variable is $\bar{y}$, $\bar{x}$ is the auxiliary mean sample auxiliary for the variable and $\bar{X}$ is the mean of the population for the auxiliary variable, and it is known and understandable. Numerous statisticians used AUI as a regression estimator, as presented by Equation (2).

$$\bar{y}_{reg} = \bar{y} + b(\bar{X} - \bar{x}) \tag{2}$$

The b is the lease square estimate of the coefficient of population regression as β. Similarly, Kadilar and Cingi (2004) utilised the auxiliary variable and recommended the ratio estimator estimate as presented in Equation (3).

$$\bar{y}_{dr} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x}}\bar{X} = \frac{\bar{y}_{reg}}{\bar{x}}\bar{X} \tag{3}$$

The respective MSE expressions of Equations (1-3) are as follows.

$$MSE(\bar{y}_r) \cong \theta\bar{Y}^2\left[C_y^2 - 2\rho C_y C_x + C_x^2\right], \tag{4}$$

$$MSE(\bar{y}_{reg}) \cong \theta\bar{Y}^2 C_y^2\left(1-\rho^2\right), \tag{5}$$

And,

$$MSE(\bar{y}_{dr}) \cong \theta\bar{Y}^2\left[C_x^2 + C_y^2\left(1-\rho^2\right)\right] \tag{6}$$

Where $\theta = \frac{1 - n/N}{n}$, n is the sample size, N is the population size, C_y is the variation coefficient for the variable of study, C_x is the auxiliary variable variation coefficient. Furthermore, S_y^2, S_x^2 are the sample inconsistency of variables both investigated and auxiliary correspondingly, and ρ is the coefficient of correlation stuck between the study and the auxiliary variables.

The rest of the research paper has the following structure: The Section 2 describes the literature review along with the research gap. The Section 3 explains the memory type estimator. Section 4 presents the proposed estimator. The Section 5 presents a numerical study related to the proposed estimator. The Section 6 discusses the study's main findings. The Section 7 illustrates an application of the proposed estimator. Concluding remarks are provided in the Section 8 of this research paper.

2. Literature review

The ratio estimate accuracy has been improved by numerous researchers, as presented in Equations (1) and (2), by amending them in dissimilar situations. Like, Sisodia and Dwivedi (1981) extended the classical ratio estimator by incorporating the variation of coefficient with the variable of auxiliary, while the extension in the estimation of Sisodia and Dwivedi (1981) was made by Upadhyaya and Singh (1999). Bahl and Nain (1999) proposed a generalised estimator motivated by Ray and Singh (1981). Kadilar *et al.* (2007) proposed estimators using robust regression. Jhajj and Walia (2012) proposed a comprehensive disparity cum estimate for ratio type in twofold sampling. A dissimilarity cum estimate of exponential type was given by Shabbir *et al.* (2014), and a modified exponential type of estimator under Simple Random Sampling (SRS) was given by Kadilar (2016). Javaid *et al.* (2019) proposed a universal estimation of the ratio for a mean of the population in the occurrence of non-response. An estimation of the ratio and product type for the known coefficient of variation via the modified maximum likelihood method was projected by Kumar and Chhaparwal (2019).

The literature review about estimates is only helpful in cross-sectional field studies. Consequently, the efficiency of the ratio estimator could be enhanced for investigations in which gathered data rely on a time scale. For such circumstances, the estimation for ratio and product with simple random type for time series statistics was proposed by Aslam *et al.* (2021), titled estimations for memory type ratio and product by using the EWMA statistic. During 2020 and 2021, memory type ratio estimations mean of the population for sampling types of stratified and ranked correspondingly was utilised by (Aslam *et al.*, 2020; 2021). Qureshi *et al.* (2022) provided estimations for the memory type ratio for variance of a population. Chhaparwal and Kumar (2022) improved the effectiveness of memory type ratio estimations using EWMA fusion. Zahid *et al.* (2023) proposed estimations for memory type ratio and product under extensive statistics of EWMA. To measure the mean, Bhushan *et al.* (2023) gave an estimate for memory type logarithm. In this manuscript, we are modifying the methodology of Noor-ul-Amin (2021) to advance the competence of estimation of the population's mean for times-scaled surveys and suggesting a new memory base ratio type estimator employing auxiliary information twice.

3. Memory type estimator

Roberts (1959) was the first to utilise the statistics of EWMA by involving the previous and existing data to improve the competence of estimates and is presented as

$$Z_t = \lambda \bar{y}_t + (1-\lambda) Z_{t-1} \qquad \text{where} \qquad t > 0 \tag{7}$$

Whereas $\bar{y}_t$ is the sample's mean at the moment in time *t*, λ is the parameter for weight and smoothing constant of clarification. Its value lies between 0-1. The bigger the importance of λ then the heavier weight is specified for the newest worth and the lesser weight to the earlier period values, while the less significant the value of λ , the superior weight is prearranged for the empirical data and small weight to the most recent data. For λ=1, the EWMA statistic becomes equal to the conventional mean of the sample. Whereas Z_{t-1} specifies the statistics of old data. The early value (Z_0) of Z_t is the anticipated mean, which may be obtained from the pilot survey (Noor-ul-Amin (2021)). The anticipated assessment and inconsistency of EWMA statistics is presented as:

$$E\left(Z_t\right)=\bar{Y} \qquad \text{and} \qquad Var\left(Z_t\right)=\frac{\sigma_y^2}{n}\left[\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2t}\right)\right],$$

where, $\bar{Y}$ and σ_y^2 is the variable of interest's mean and variance. The restrictive type of variance is illustrated by Equation (8).

$$Var\left(Z_t\right)=\frac{\sigma_y^2}{n}\left[\frac{\lambda}{2-\lambda}\right]. \tag{8}$$

The proposed estimate for the memory type ratio of Noor ul Amin (2021) is given by Equation (9).

$$y_{rt}^{M}=\frac{Z_{yt}}{E_{xt}}\bar{X}, \tag{9}$$

Where $Z_{yt}=\lambda\,\bar{y}_t+(1-\lambda)Z_{y(t-1)}$, $E_{xt}=\lambda\,\bar{x}_t+(1-\lambda)E_{x(t-1)}$ are the statistics of memory type for study and auxiliary variable, respectively.

4. Proposed memory type estimator

In this section, we proposed memory based estimator by modifying the estimator suggested by Kadilar and Cingi (2004) for time scale survey. The EWMA statistic, as given in Equation (7), is utilized to propose an estimation of memory type. For the projected estimation, the EWMA statistic at time *t* for study variable (*y*), represented by E_{yt} is obtained by using the regression estimator in Equation (7) and is given below:

$$E_{yt}=\lambda\,\bar{y}_{regt}+(1-\lambda)E_{y(t-1)} \tag{10}$$

Where $\bar{y}_{regt}$ is given in Equation (2) at time *t* and EWMA statistic for variable of auxiliary (*x*) signified as E_{xt} such that.

$$E_{xt}=\lambda\,\bar{x}_t+(1-\lambda)E_{x(t-1)} \tag{11}$$

Where $\bar{x}_t$ is the mean per unit estimate under SRS plan at period *t*. The planned estimation for memory type ratio in SRS is designed as follows:

$$\bar{y}_{drt}^{M} = \frac{E_{yt}}{E_{xt}} \bar{X} , \tag{12}$$

The estimated expression of MSE for the projected estimate of the ratio is presented by Equation (13).

$$\text{MSE}\left(\bar{y}_{drt}^{M}\right) \approx \theta \bar{Y}^2 \frac{\lambda}{2-\lambda}\left[C_y^2\left(1-\rho^2\right)+C_x^2\right] \tag{13}$$

(Detailed proof of MSE is shown in the Appendix).

5. Numerical study

A broader statistical investigation is done to assess the efficiency of the estimation under study. The presented MSEs are founded on fifty thousand repeated attempts to project estimation. The MSE is measured for the proposed estimator by utilising the method specified in Equation (14).

$$MSE\,(a) = \frac{1}{50,000} \sum_{i=1}^{50000} (a_i - \bar{Y})^2 \tag{14}$$

Where $a = \bar{Y}_{rt}, \bar{Y}_{drt}, \bar{Y}_{rt}^{M}, \bar{Y}_{drt}^{M}$ and μ_a is the mean with 50,000 samples. Equation (15) provides the Relative Efficiencies (REs) for mean per unit estimates.

$$RE\,(a) = \frac{MSE\,(\bar{Y}_t)}{MSE\,(a)}, \tag{15}$$

The projected MSE values are given in Table-1, and the outcomes of REs are presented in Table-2. The outcomes of MSE and REs are obtained for various values of coefficient correlations, i.e. 0.25, 0.50, 0.75, .95. To observe the effect of smoothing constant, different values of λ , like .05, .10, .25, .50, .75, 1.0 have been used.

Subsequent instructions have been used to measure the values of MSEs and REs for the projected ratio of memory type with SRS.

- A populace size of 10000 is generated from a distribution of bivariate normal with parameters $(Y, X) \sim N_2(1, 50, 1, 1, \rho)$.
- Select the random sample of size *n*=10-30, 50, 200, and 500.
- Calculate the statistics of EWMA presented in (10-11) with diverse options for smoothing constants.
- Measure the estimations of the ratio presented in Equation (12).
- Replicate steps (i) to (iv) for fifty thousand times.
- The projected MSE is measured using formula given in Equation (14), and the outcomes are formed in Table-1.

- We got REs for each size of sample size with the application of Equation (15), which is given in Table-2.

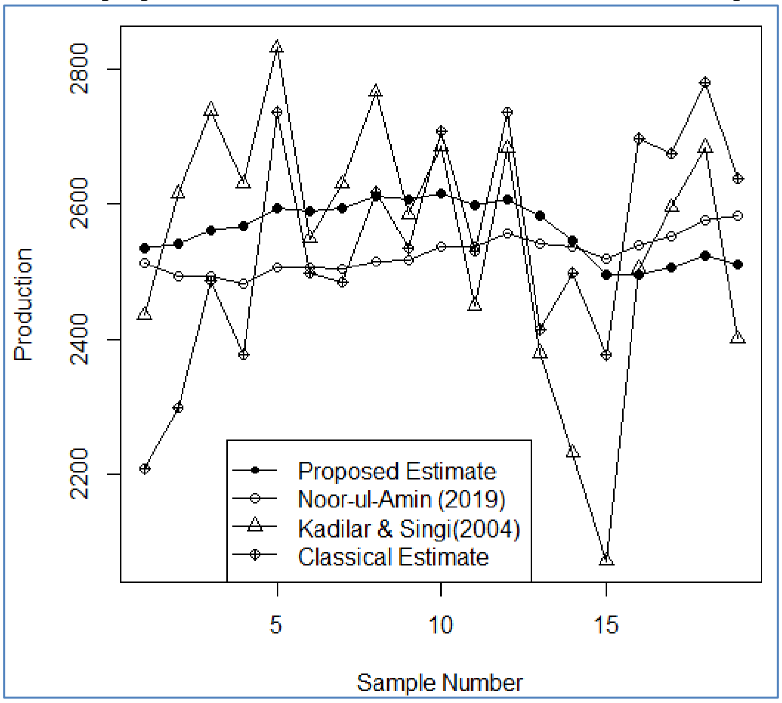
6. Main findings

The premeditated outcomes of MSEs and REs for the projected type are illustrated in 1 and 2 Tables. The proportional investigation for projected MSE and REs of memory type estimate with twice application of AUI placed on SRS phase are presented in Tables-1 and 2 and the foremost conclusions of the proposed estimation are:

- It is prominent in Table-1 that MSEs of the projected estimate of memory type given in (12) are smaller than the estimator given in (1), (3) and (9). Consider for example, for λ = 0.05, ρ = 0.75 and n = 30, MSE of (1) is 0.0324, (3) is 0.0147, (9) is 0.0009 and the proposed estimator (12) is 0.0004. This shows that the proposed estimator (12) performs better than (1), (3), and (9).
- The REs of the planned estimate is given Table-2, these REs are higher than (1), (3), and (9). Consider for example, for λ = 0.50, ρ = 0.75 and n = 50, RE of (1) is 1.0306, (3) is 2.3003, (9) is 3.0791, and the proposed estimator (12) is 6.7745. This establishes the competence of the planned ratio estimation with respect to classical and other corresponding ratio estimators.
- As ρ i.e. coefficient of correlation among variable of study and auxiliary rises from .25 to .95, the figures of MSEs shift downward to augment the competency of the planned estimation (12). For example, consider λ= 0.50, n = 50, as ρ increases from 0.50 to 0.75, and then for (1), MSE decreases from 0.0195 to 0.0194, with corresponding RE increases from 1.0199 to 1.0306. For (3), MSE decreased from 0.0149 to 0.0087, and the corresponding RE increased from 1.3343 to 2.3003. For (9), MSE decreased from 0.0066 to 0.0065 with corresponding RE increases from 3.0644 to 3.0791 and for (12), MSE decreased from 0.0051 to 0.0030 with corresponding RE increases from 3.9723 to 6.7745. Increase in ρ From 0.50 to 0.75, the decrease in MSE for (1), (3), (9), and (12) is 0.0001, 0.0062, 0.0001, and 0.0021, respectively, with an increase in the corresponding RE of 0.0107, 0.9660, 0.0147, and 2.8022. This shows that the proposed estimator excels at performance compared to (1), (3), and (9). Hence, the utilization of AUI enlarges the effectiveness of the estimate.
- For unchanging lambda value and correlation coefficient, as the size of sample enlarges, it means that *n* opt values 10, 20, 30, 50, 100, 200, 500, the MSEs decreased with each improved value of the size of sample e.g. consider λ = 0.05, ρ = 0.75, as *n* increases from 30 to 50, the corresponding MSE decreases from 0.0324 to 0.0195, 0.0147 to 0.0087, 0.0009 to 0.0005, and 0.0004 to 0.0002 for (1), (3), (9), and (12), respectively.
- To use the previous outcomes, align with the existing information, a smoothing constant λ is used. For example, fix ρ = 0.75, n = 30 as λ decreases from 0.25 to 0.10, then MSE decreases from 0.0047 to 0.0017 and 0.0021 to 0.0008 for (9) and (12), respectively. This shows that as λ decrease, weight for past information increase and weight decrease for current information. This can guide the competence of the anticipated estimate and is given in Tables-2. Finally, $\lambda = 1$ shows that only the existing data is used and that there is no consumption of previous data. This can be seen from the last two columns of table-2 that the EWMA-based planned estimation will rely upon recent information only to

approximate the mean just like (3). Hence, the performance of the (12) will be equally suitable for $\lambda = 1$.

Figure 1: Comparison of proposed estimator with other ratio estimators for the production of wheat



7. Conclusion

During a time-scaled field study, utilising the existing and previous information about the sample is significant. To handle such a problem, the Noor-ul-Amin model helped to use ratio estimates by applying the statistics of EWMA for the cross-sectional field studies. In the current study, we try to improve the competence of the model mentioned above by double-using auxiliary information differently. We scrutinise the projected estimate by doing an extensive investigation. It is concluded from the numerical research and actual life use that the projected estimation showed better outcomes for error of mean square and efficiency of relativism than the proportional estimate. Therefore, it is further concluded that the anticipated memory type estimation under SRS is more well-organized than their comparative estimations deliberated and shown in the numerical investigation.

Declaration of conflict of interest

The author(s) declared no potential conflicts of interest(s) with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship and/or publication of this article.

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Appendix:

In order to derive the MSE of the proposed estimator, we define the following notations such as.

$e_{yt} = \frac{E_{yt} - \bar{Y}}{\bar{Y}}$ and $e_{xt} = \frac{E_{xt} - \bar{X}}{\bar{X}}$

$E(e_{yt}) = 0, \; E(e_{xt}) = 0,$

$$E(e_{yt}^2) = \frac{Var(E_{yt})}{\bar{Y}^2} = \frac{1}{\bar{Y}^2}\left[\frac{\lambda}{2-\lambda}\right] Var(\bar{y}_{regt})$$

Further we have,

$$E(e_{yt}e_{xt}) = \frac{1}{\bar{X}\bar{Y}} Cov(E_{yt}, E_{xt}) = \frac{1}{\bar{X}\bar{Y}}\left[\frac{\lambda}{2-\lambda}\right] Cov(\bar{y}_{regt}, \bar{x}_t),$$

Where $Var(\bar{y}_{regt}) = \theta \frac{S_y^2}{n}(1-\rho^2)$, $Var(\bar{x}_t) = \theta \frac{S_x^2}{n}$ and $Cov(\bar{y}_{reg}, \bar{x}_t) = 0$.

To derive the MSE expression put the value of E_{yt} and E_{xt} into (12) and then simplifying by Taylor series approximation, we have.

$$\bar{y}_{drt}^M = \bar{Y}(1+e_{yt})(1+e_{xt})^{-1},$$

Further simplifying and ignoring the higher-order terms, the approximate expression is obtained as follows.

$$\bar{y}_{drt}^M \approx \bar{Y}(1+e_{yt})(1-e_{xt}),$$

The approximate mean square error expression is given by.

$$MSE(\bar{y}_{drt}^M) \approx \bar{Y}^2\left[Var(E_{yt}) + R^2 Var(E_{xt}) - 2R\,Cov(E_{yt}, E_{xt})\right].$$

$$\mathrm{MSE}(\bar{y}_{drt}^M) \approx \frac{\lambda}{2-\lambda}\left[\mathrm{Var}(\bar{y}_{regt}) + R^2 \mathrm{Var}(\bar{x}_t) - 2R\,\mathrm{Cov}(\bar{y}_{reg}, \bar{x}_t)\right]$$

$$\mathrm{MSE}(\bar{y}_{drt}^M) \approx \theta \frac{\lambda}{2-\lambda}\left[S_y^2(1-\rho^2) + R^2 S_x^2\right]$$

$$\mathrm{MSE}(\bar{y}_{drt}^M) \approx \theta \bar{Y}^2 \frac{\lambda}{2-\lambda}\left[C_y^2(1-\rho^2) + C_x^2\right]$$

Table-1: MSEs of proposed and under study estimators

				$\lambda = 0.05$		$\lambda = 0.10$		$\lambda = 0.25$		$\lambda = 0.50$		$\lambda = 0.75$		$\lambda = 1.0$	
ρ	n	$\bar{y}_{rt}$	$\bar{y}_{drt}$	$\bar{y}^{M}_{rt}$	$\bar{y}^{M}_{drt}$	$\bar{y}^{M}_{rt}$	$\bar{y}^{M}_{drt}$	$\bar{y}^{M}_{rt}$	$\bar{y}^{M}_{drt}$	$\bar{y}^{M}_{rt}$	$\bar{y}^{M}_{drt}$	$\bar{y}^{M}_{rt}$	$\bar{y}^{M}_{drt}$	$\bar{y}^{M}_{rt}$	$\bar{y}^{M}_{drt}$
0.25	10	0.0992	0.0940	0.0025	0.0024	0.0051	0.0048	00144	0.0136	0.0324	0.0308	0.0598	0.0564	0.0992	0.0940
	20	0.0491	0.0464	0.0013	0.0012	0.0025	0.0024	0.007	0.0066	0.0163	0.0154	0.0297	0.0281	0.0491	0.0464
	30	0.0333	0.0317	0.0008	0.0008	0.0018	0.0017	0.0047	0.0044	0.0110	0.0104	0.0198	0.0187	0.0333	0.0317
	50	0.0199	0.0188	0.0005	0.0005	0.0011	0.001	0.0029	0.0027	0.0066	0.0062	0.0119	0.0113	0.0199	0.0188
	200	0.0050	0.0048	0.0001	0.0001	0.0003	0.0002	0.0007	0.0007	0.0016	0.0016	0.0030	0.0028	0.0050	0.0048
	500	0.0020	0.0019	0.0000	0.0000	0.0001	0.0001	0.0003	0.0003	0.0007	0.0006	0.0012	0.0011	0.0020	0.0019
0.50	10	0.0985	0.0753	0.0025	0.0020	0.0050	0.0040	0.0143	0.0109	0.0329	0.0252	0.0593	0.0452	0.0985	0.0753
	20	0.0492	0.0376	0.0013	0.0010	0.0026	0.0020	0.0071	0.0055	0.0163	0.0125	0.0295	0.0226	0.0492	0.0376
	30	0.0327	0.0250	0.0009	0.0007	0.0018	0.0013	0.0046	0.0036	0.0109	0.0083	0.0197	0.0151	0.0327	0.0250
	50	0.0195	0.0149	0.0005	0.0004	0.0010	0.0008	0.0028	0.0021	0.0066	0.0051	0.0117	0.0090	0.0195	0.0149
	200	0.0049	0.0038	0.0001	0.0001	0.0003	0.0002	0.0007	0.0005	0.0016	0.0012	0.0030	0.0023	0.0049	0.0038
	500	0.0020	0.0015	0.0001	0.0000	0.0001	0.0001	0.0003	0.0002	0.0007	0.0005	0.0012	0.0009	0.0020	0.0015
0.75	10	0.0969	0.0433	0.0024	0.0011	0.0052	0.0023	0.0141	0.0063	0.0324	0.0147	0.0584	0.0264	0.0969	0.0433
	20	0.0494	0.0220	0.0013	0.0006	0.0025	0.0011	0.0071	0.0032	0.0163	0.0073	0.0295	0.0133	0.0494	0.0220
	30	0.0324	0.0147	0.0009	0.0004	0.0017	0.0008	0.0047	0.0021	0.0108	0.0049	0.0195	0.0087	0.0324	0.0147
	50	0.0194	0.0087	0.0005	0.0002	0.0010	0.0005	0.0028	0.0013	0.0065	0.0030	0.0115	0.0052	0.0195	0.0087
	200	0.0048	0.0022	0.0001	0.0001	0.0003	0.0001	0.0007	0.0003	0.0016	0.0007	0.0029	0.0013	0.0048	0.0022
	500	0.0019	0.0009	0.0000	0.0000	0.0001	0.0000	0.0003	0.0001	0.0006	0.0003	0.0012	0.0005	0.0019	0.0009
0.95	10	0.0974	0.0100	0.0025	0.0002	0.0050	0.0005	0.0137	0.0014	0.0319	0.0032	0.0577	0.0059	0.0974	0.0100
	20	0.0487	0.0049	0.0013	0.0001	0.0025	0.0003	0.0069	0.0007	0.0162	0.0016	0.0293	0.0029	0.0487	0.0049
	30	0.0323	0.0033	0.0009	0.0001	0.0018	0.0002	0.0045	0.0005	0.0107	0.0011	0.0194	0.0020	0.0323	0.0033
	50	0.0191	0.0020	0.0005	0.0000	0.0010	0.0001	0.0027	0.0003	0.0065	0.0006	0.0115	0.0012	0.0191	0.0020
	200	0.0048	0.0005	0.0001	0.0000	0.0003	0.0000	0.0007	0.0001	0.0016	0.0002	0.0029	0.0003	0.0048	0.0005
	500	0.0019	0.0002	0.0001	0.0000	0.0001	0.0000	0.0003	0.0000	0.0006	0.0001	0.0011	0.0001	0.0019	0.0002

Note: *0.0000 is approximately zero up to 4 decimal places, but RE is possible to calculate.

Table-2: REs of proposed and under study estimators

				λ = 0.05		λ = 0.10		λ = 0.25		λ = 0.50		λ = 0.75		λ = 1.0	
ρ	n	$\bar{y}_{rt}$	$\bar{y}_{drt}$	$\bar{y}_{rt}^{M}$	$\bar{y}_{drt}^{M}$	$\bar{y}_{rt}^{M}$	$\bar{y}_{drt}^{M}$	$\bar{y}_{rt}^{M}$	$\bar{y}_{drt}^{M}$	$\bar{y}_{rt}^{M}$	$\bar{y}_{drt}^{M}$	$\bar{y}_{rt}^{M}$	$\bar{y}_{drt}^{M}$	$\bar{y}_{rt}^{M}$	$\bar{y}_{drt}^{M}$
0.25	10	1.0094	1.0644	39.4730	41.7439	19.4426	20.6331	6.9729	7.3696	3.0722	3.2264	1.6800	1.7818	1.0094	1.0644
	20	1.0099	1.0697	39.1942	42.3537	19.3918	20.4944	7.0193	7.4407	3.0523	3.2269	1.6882	1.7845	1.0099	1.0697
	30	1.0092	1.0597	40.4996	42.2205	18.6895	19.6392	7.0966	7.4998	3.0408	3.2202	1.6874	1.7911	1.0092	1.0597
	50	1.0099	1.0707	40.5258	42.8797	19.0096	19.908	6.9748	7.4021	3.0064	3.1882	1.6891	1.7824	1.0099	1.0707
	200	1.0095	1.0653	39.1855	41.4189	19.4524	20.5757	7.1107	7.5424	3.0299	3.1901	1.6759	1.7702	1.0095	1.0653
	500	1.0092	1.0595	40.3221	42.6533	19.1295	20.203	7.0964	7.5086	2.9931	3.1659	1.6831	1.7835	1.0092	1.0595
0.50	10	1.0198	1.3334	39.7530	50.4780	19.6894	24.9891	7.1098	9.3289	3.0421	3.9797	1.6994	2.2314	1.0198	1.3334
	20	1.0201	1.3327	38.4063	49.3074	19.4933	25.3853	7.0421	9.2197	3.0644	3.9922	1.6985	2.2128	1.0201	1.3327
	30	1.0198	1.3336	38.9980	50.5988	18.8443	25.2183	7.1627	9.3389	3.0739	4.0315	1.7033	2.2140	1.0198	1.3336
	50	1.0199	1.3343	39.9198	52.0882	19.4983	25.6170	7.0970	9.3464	3.0644	3.9723	1.7017	2.1972	1.0199	1.3343
	200	1.0197	1.3258	40.5124	54.5797	19.9137	25.9982	7.1460	9.1687	3.0791	4.0100	1.6954	2.2318	1.0197	1.3258
	500	1.0198	1.3298	39.3909	51.1535	18.9841	24.4014	7.2072	9.6132	3.0728	4.0028	1.6997	2.2119	1.0198	1.3298
0.75	10	1.0302	2.3029	41.4895	86.6630	19.0788	43.3361	7.1626	16.0867	3.0960	6.8528	1.7147	3.7920	1.0302	2.3029
	20	1.0304	2.3118	38.5036	88.3619	19.7758	43.7049	7.0465	15.7400	3.0832	6.8727	1.7141	3.8062	1.0304	2.3118
	30	1.0305	2.2702	38.8771	82.7091	19.2906	44.0687	7.2424	16.2949	3.0988	6.8239	1.7177	3.8399	1.0305	2.2702
	50	1.0306	2.3003	40.7016	86.5807	19.3094	41.6407	7.0624	15.9332	3.0791	6.7745	1.7231	3.7861	1.0306	2.3003
	200	1.0306	2.2783	40.8476	91.5270	19.4387	43.8971	7.1738	15.7367	3.1067	6.7929	1.7181	3.7814	1.0306	2.2783
	500	1.0307	2.2948	40.6645	90.0121	19.1324	43.0893	7.1639	15.8451	3.0908	6.8845	1.7137	3.7979	1.0307	2.2948
0.95	10	1.0388	10.1680	39.1896	412.7123	19.8061	201.2009	7.2850	71.1810	3.1318	30.7234	1.7302	16.8825	1.0388	10.1680
	20	1.0390	10.3442	39.3882	384.9747	19.8865	196.3518	7.3471	70.2270	3.0968	30.4949	1.7255	17.2873	1.0390	10.3442
	30	1.0389	10.2209	38.5888	400.7982	19.2704	195.2936	7.3278	70.0952	3.1146	30.7048	1.7283	17.1234	1.0389	10.2209
	50	1.0389	10.0618	39.4057	402.6739	20.6132	192.8753	7.2892	71.5061	3.1178	31.1615	1.7341	16.9859	1.0389	10.0618
	200	1.0390	10.1426	41.9369	416.5830	19.7076	194.2038	7.3832	70.7520	3.1124	30.6114	1.7287	16.9016	1.0390	10.1426
	500	1.0390	10.0827	39.9042	384.8493	19.9536	191.9870	7.2645	72.0274	3.1156	30.2129	1.7288	16.9638	1.0390	10.0827

Table-3: Summary results of illustrative example

Crop Year	Simple Mean		EWMA mean			Mean Estimator			
	$\bar{y}_t$	$\bar{x}_t$	Z_{yt}	E_{yt}	E_{xt}	$\bar{y}_{rt}$	$\bar{y}_{drt}$	$\bar{y}_{rt}^{M}$	$\bar{y}_{drt}^{M}$
1996	3429.1	1975.6	2498.979	2520.347	6304.458	2209.107	2435.947	2513.555	2535.048
1997	3443.2	1977.6	2460.881	2509.245	6258.003	2299.822	2616.149	2493.610	2542.617
1998	3339.5	1998.5	2447.493	2514.649	6225.662	2486.443	2738.923	2492.927	2561.329
1999	3315.2	1998.5	2425.344	2509.413	6196.556	2378.523	2631.003	2481.970	2568.002
2000	2918.9	1863.2	2449.509	2534.523	6194.93	2736.447	2832.398	2507.358	2594.379
2001	3486.3	2065.6	2451.058	2532.553	6200.987	2498.783	2549.283	2506.493	2589.830
2002	3240.9	1963.0	2445.153	2532.411	6191.069	2485.861	2630.452	2504.459	2593.834
2003	3200.2	2041.7	2452.437	2545.144	6181.692	2618.736	2766.153	2515.731	2610.830
2004	3220.1	2106.9	2457.194	2545.612	6189.073	2534.262	2584.763	2517.604	2608.196
2005	3260.8	2112.7	2483.874	2561.262	6208.055	2707.914	2686.155	2537.159	2616.206
2006	3243.1	2096.9	2494.287	2555.674	6235.590	2531.250	2450.449	2536.544	2598.972
2007	3116.1	1973.4	2522.358	2572.286	6255.311	2735.499	2683.047	2557.004	2607.618
2008	3994.7	2187.1	2513.922	2555.326	6269.98	2414.858	2379.882	2542.490	2584.365
2009	3446.5	2124.6	2531.930	2540.431	6326.602	2498.944	2232.147	2537.785	2546.306
2010	3966.5	2161.8	2537.937	2512.334	6385.292	2377.446	2072.433	2520.425	2494.999
2011	4304.8	2251.1	2568.743	2525.376	6415.862	2697.227	2504.605	2538.863	2496.000
2012	4333.0	2317.3	2585.469	2538.206	6422.566	2676.211	2595.688	2552.727	2506.063
2013	4673.8	2423.6	2612.422	2560.003	6431.440	2780.430	2684.187	2575.780	2524.096
2014	5155.6	2515.4	2633.280	2560.699	6466.136	2639.061	2401.411	2582.414	2511.235