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Original Research Article

Exploring the fundamental properties and MGF of a 3-component mixture of power distributions: a simulation study

Umar Waqas[1] | Nazakat Ali*[2] | Hafiz Bilal Ahmad[2] | Amir Shahzad[2] | Suhail Ashraf [3]

1. Department of Statistics, G. C. University Faisalabad, Faisalabad, Pakistan.
2. Department of Mathematics and Statistics, University of Agriculture, Faisalabad, Pakistan.
3. School of Mathematics, Computer Science and Engineering, Liverpool Hope University, UK.

*Corresponding Author Email: nazakatalibandesha@gmail.com

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Abstract: This paper estimates the parameters of the 3-component mixture of power distributions. The expressions of fundamental properties of the said distribution are presented, and for judgment, the numerical results of mean, median, variance, mode and skewness are evaluated. The closed-form expressions of the reliability properties, the statistical properties and necessary inequality measures are discussed. The trend of reliability measures is shown with the help of numerical results and graphs by taking different values of parameters and proportion parameters. The mathematical expressions of certain related statistical functions are derived from the reliability functions, the moment generating function, the characteristics function, the probability generating function, and the factorial moment generating function. The expressions of essential entropies, Shannon's entropy, the β -entropy and Renyi's entropy, are derived. The mathematical expressions of inequality indexes, the Gini index, the Lorenz curve, the Bonferroni curve, the Zinga index, the Atkinson index and the generalized entropy index are also analysed. The RF, the HRF, the CHR, the RHR, the MRL, and the MWT functions are derived in reliability analysis. The numerical values and graphs for different parameters and proportion components are discussed to judge the behaviour of reliability properties.

Keywords: Moment generating function, Reliability properties, Statistical properties, Statistical functions, Entropies, Inequality measures, CHR function, RHR function.

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1. Introduction

A mixture distribution arises naturally when a statistical population includes several subpopulations. This classification has gained considerable interest in a variety of applications, particularly in the field of performance improvement and reliability studies. A mixture distribution involves a combination of probability distributions, each representing a subpopulation, with a corresponding weight called the mixture weight. If population sizes are unknown, the standard distribution may not apply, making finite mixture models with appropriate probability distributions necessary e.g., electrical component or chemical concentrations may be subpopulations defined by failures. The hybrid distribution can be either discrete or continuous depending on the number of parents and is useful when multiple scenarios are possible but lack context-specific information.

When the data from individual component densities or the conditional distributions are not available, but these were available from an overall mixture distribution then the necessary use becomes the finite mixture model. When the mixture data is modelled of some of the element distributions then mixture models of straight application is obtained. Otherwise, it is said to be an indirect application, for example, the cluster analysis, the kernel-based density estimation and the modelling prior densities. Mostly, in the biology the direct applications of the mixture models are discussed by (Bhattacharya, 1967) and by (Gregor, 1969), in the medicine are presented by (Chivers, 1977) and by (Burckhardt, 1978), in the social sciences are observed by Harris in 1983, in an economics are mentioned by (Jedidi et al., 1997), in the reliability and survival are analysed by (Sultan et al., 2007), in the life testing are suggested by Shawky and (Shawky & Bakoban, 2009), in the industrial engineering are observed by (Ali et al., 2012), etc. In the recent years, there are various applied fields in which a mixture representation has been recycled is still spreading are due to a rapid growth of the powerful computational techniques. It is clear in many applications; that the available data comes from a mixture of more than two distributions. Then these applications enable us to combination the statistical allocations for getting a new mixture distribution. To study a population some of an appropriate probability distribution is proposed in a mixture that is imaginary to depends on several subpopulations which mixed in the unknown proportions.

In practical situations, a finite population of mixtures has determined the distribution of components, allowing focus on mixture theory. Sometimes component distributions are known with unknown parameters, while mixing ratios are determined. This thesis examines classical research on 3-component mixtures of Power distributions, focusing on unknown parameters and proportions under the Type-I mixture model.

If X has the following density function can be written in the form $f(x) = \sum_{i=1}^{k} w_i \, f_i(x)$, where w_i $(i = 1, 2, ..., K)$ is i^{th} mixing proportions such that $w_k = 1 - \sum_{i=1}^{k-1} w_i$ and $f_i(x)$ is i^{th} component density function then X random variable is supposed a finite mixture of distribution to follow with a k components. With unknown mixing proportions w_1 and w_2, the Probability Density Function (PDF) and Cumulative distribution function (cdf) of the finite three-component mixture of distribution is defined by the (Barger et al., 2006) and by (Stehlík et al., 2012) as:

$$f(x; \Psi) = w_1 f_1(x; \beta_1) + w_2 f_2(x; \beta_2) + (1 - w_1 - w_2) f_3(x; \beta_3), \qquad 0 < x < 1 \tag{1}$$

$$F(x; \Psi) = w_1 F_1(x; \beta_1) + w_2 F_2(x; \beta_2) + (1 - w_1 - w_2) F_3(x; \beta_3) \tag{2}$$

Where $w_i \geq 0, \sum_{i=1}^{2} w_i \leq 1, \Psi = (\beta_1, \beta_2, \beta_3, w_1, w_2)$ and, $f_i(x; \Psi_i), i = 1, 2, 3$ is the PDF of the i^{th} component.

Modern data analysis uses various types of data, such as simple data, aggregate data, censored data, progressively censored data, and value data. Censorship is a common problem in life sciences, which occurs when only part of the information about the source of information is known. Censorship is a property of the dataset, not a parameter, and is different from truncation. While truncation involves not including data outside of a certain data, in censoring, the value must be known after a period of time, but the truth must not be known. The main analysis methods are right analysis, left analysis, and periodic analysis. The review policy includes type 1 and type 2. Type 1 is divided into ordinary type, advanced type and general type. This study focuses on the ordinary type I censorship law where the cut-off time is fixed. Previous studies have investigated various censoring methods, including (Sindhu *et al.*, 2015) and others.

Reliability analysis is used to measure time-to-failure data with consistent results demonstrating reliability under similar conditions. Mixed distributions are popular for modelling heterogeneous data. Chen *et al.* (1985) used a mixed model, first popularized by Berkson and Gage (1952), for cancer survival data. Quiang (1994) used a similar model with a Weibull component for cancer testing. Rahnama *et al.* (2006) used a multivariate model for cancer progression with significant survival as covariates. Bayesian estimation of exponential survival time was not investigated by Abu-Taleb *et al.* (2007). Erişoğlu *et al.* (2011) analysed the heterogeneity of survival profiles in two distributions. Krishna and Malik (2012) proposed reliability estimates for the Maxwell distribution using asymptotic type II censoring, while Ali (2014) provided reliable properties for two-point mixtures of Rayleigh returns.

The primary objective of this study is the classical estimation of a three-component mixture of Power distributions. Additionally, the study aims to conduct a comprehensive reliability analysis of these mixtures. Other key objectives include developing the analytical form of the three-component mixture of power distributions, deriving the basic statistical properties, and obtaining closed-form mathematical expressions for various functions related to reliability analysis. The study also seeks to establish algebraic expressions for the entropies of the three-component mixture and to determine the expressions for various inequality measures associated with these distributions.

2. Literature of review

Ali *et al.* (2005) discussed the characterization of Power distribution. They worked on the record statistics of the mixture of two-exponential distributions by estimating the parameters. ML and Bayes methods of estimation are used for estimating parameters. In 1980, Lindley used the approximation form to obtain Bayes estimators for parameters of mixture model. By using Monte Carlo simulation method, Bayes estimates results are compared with estimates obtained by the method of ML.

Soliman *et al.* (2010) focused on Bayesian analysis of a mixture of power functions by comparing Bayesian and non-Bayesian estimation methods for unknown parameters based on lower cost. They produce Bayesian estimates in zero-sum squared error loss derived using prior data. This work also presents Bayesian time estimation and time estimation exemplified by valid models.

Afify (2011) introduced the compound Rayleigh distribution and derived ML estimates of its parameters, including the asymptotic variance-covariance matrix. This study addresses the problem of estimation under type 1 asymptotic censoring and tests the new estimates with numerical examples. A Monte Carlo simulation study investigated the accuracy of the ML estimator and the iteration process.

Tahir *et al.* (2016) the study examined factors, and parameter estimates of three-factor mixtures of exponential distributions (3-CMED) using the maximum likelihood (ML) method and discusses the main properties of statistical reliability, entropy measures, dissimilarity indices, and order statistics. Parameter estimates are performed through ML, using both censored and fully censored sampling, validated by simulation studies on real-life data.

Aslam *et al.* (2018) developed the reliability analysis of 3-CMED, 3-CMRD, 3CMBD and 3-CMPD. The problems of evaluating different survival time properties of 3-CMED, Rayleigh, Pareto and Burr distributions are developed. For these 3-CMD, the CDF, HRF, MRL, MWT functions and numerical results are derived for fixed standards of factors.

Kulagina (2022) used Mixture models in various applications such as clustering, random effects modelling, and divergence, modelling heterogeneous populations that are homogeneous subpopulations Estimating the number of factors in a mixture model is a challenging task. Methods such as Hankel-matrix methods, minimum distance estimation, and likelihood ratio tests, all with consistency guarantees, provide promising but infallible solutions, with useful implications for specific data situations.

Hou *et al.* (2023) overhead power distribution infrastructure is vulnerable to ice storms, causing significant economic losses each year. Climate change and aging infrastructure are increasing the need for resilient systems. This paper presents a possible framework for assessing the resilience of power distribution systems to ice storms, focusing on a simple model of the components involved and testing mechanisms such as tree height and vegetation management research.

According to Dey et al (2023), the research gap in this study lies in the limited exploration of alternative parameter estimation methods beyond Maximum Likelihood (ML), particularly under censored data conditions. While the paper focuses on ML estimators, their limitations in handling complex data structures, especially with censoring, are not fully addressed. Additionally, the study does not compare ML with other estimation techniques such as Bayesian methods or robust estimation, which could provide deeper insights into improving the accuracy and reliability of parameter estimation. Further investigation into these methods, especially in the context of order statistics and mixture distributions, is warranted to address this gap.

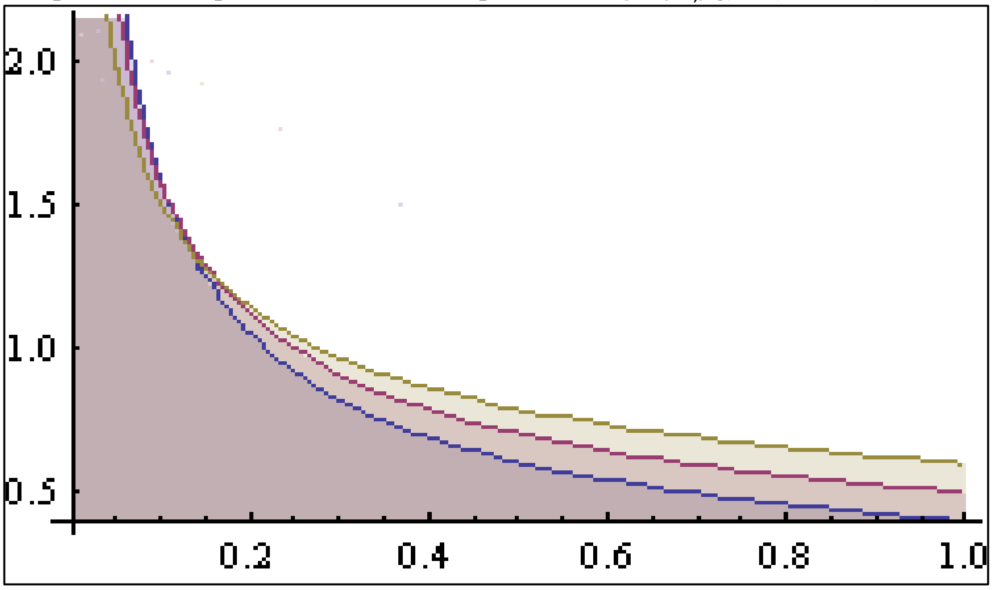
Although the ability of hybrid models to handle heterogeneous lifetime scenarios has been studied, there are few studies on the application of these models to generalized Rayleigh distributions, especially classical and Under Bayesian methods. This paper addresses the gap by introducing a more flexible hybrid Rayleigh distribution, capable of handling complex lifetime scenarios with different failure rate behaviours Both frequentist and Bayesian approaches are used for parameter estimation further extends the analysis, but comparative studies of these methods have been well studied.

3. Methodology

3.1. Power distribution

Analysis is important in life cycle data analysis because it is often impractical to track the entire duration of each activity in the data set. Censoring occurs when at least one product only partially represents the time to failure and can be divided into right censoring, left censoring, and time censoring. Right censoring can be Type I or Type II, depending on the number of failures or whether time is constant. A combination of probabilities is used to examine populations by group. As discussed, for example, by Davis (1952) and Epstein and Sobel, (1953), the life of electrical equipment can vary for many reasons. Bayesian analysis of the power function distribution, as investigated by (Meniconi & Barry, 1996), is still limited in the literature. For various values of the component parameters the behaviour of the power distribution is shown in Figure 1.

Figure 1: Graph of PDF of power distribution for parameters $(\beta_1, \beta_{2,}\beta_3)$ = (0.5, 1, 2)



The uses of power distribution are as follows:

- In communications theory power distribution is used, to describe the several paths of the scattered dense signals which reached the receiver.
- In physical sciences power distribution is used to observe the speed of wind, sound or light radiation and the wave heights.
- In an engineering, Power distribution is used, to note down an object's lifetime, where the object's lifetime based on the age of an object: e.g.: the resistors, the transformers, and the capacitors in sets of an aircraft radar.
- In medical imaging science power distribution is used, to demonstrate the noise variation in the imaging of magnetic resonance.

3.2. Statistical properties

Some of the basic properties of a statistical distribution are given below:

3.2.1. Mean and variance

Mean and variance refers to the measures of location and the measure of variation, respectively, moreover of a probability distribution or of a random variable considered by that distribution.

Mean and variance for a X r.v. is defined as:

$$E(X) = \int_0^1 xf(x)dx \tag{3}$$

$$Var(X) = E(X^2) - \{E(X)\}^2 \tag{4}$$

3.2.2. r^{th} moments about origin

The set of parameters that review it known as the moments of a distribution. In other words, the factors (constants) of a population to finding the mean, variance etc. are called the moments. The population characteristics are decided with the help of these factors and discussion of a population is based on these characteristics. The arithmetic mean, variance and standard deviation are obtained directly from moments. Moments are applicable in finding the dispersion, the averages, the kurtosis and skewness of a distribution.

The r^{th} moment about origin of X r.v. is given in Equation (5) as:

$$E(X^r) = \int_0^1 x^r f(x)dx. \tag{5}$$

The r^{th} moments of origin are also known as the raw moments.

3.2.3. h^{th} order negative moments

The h^{th} order -ve moments for a random variable X are calculated by interchanging r with $(-h)$ in Equation (5) as:

$$E(X^{-h}) = \int_0^1 x^{-h} f(x)dx. \tag{6}$$

3.2.4. Factorial moments

The factorial moments are a quantity of mathematics which defined as: The average of the dropping factorial of an arbitrary variable. To study the integer of non-negative random variables the Factorial moments are used, and these are also used for the derivation of random variables which are discrete in the probability generating functions it arises.

The factorial moments for a X random variable are derived by this relation as follows:

$$E(X(X-1)(X-2)\ldots(X-\alpha+1)) = \sum_{u=0}^{\alpha-1} \xi_u (-1)^u E(X^{\alpha-u}), \tag{7}$$

Where "$\xi_u^{'}s$" are the non-null real numbers.

The $E(X^{\alpha-\mu})$ can be obtained by replacing r with $(\alpha-\mu)$ in Equation (5) as:

$$E(X^{\alpha-\mu}) = \int_0^1 x^{\alpha-\mu} f(x)dx \qquad (8)$$

3.2.5. Quantile function

The quantile function requires, the rate at which a probability of arbitrary variable is fewer than or equivalent to the assumed probability, for the assumed probability in a probability distribution of an arbitrary variable. The quantile function is also known as the function of percent point or a function of an inverse cumulative distribution. For the value of x the function of quantile Q in positions of a distribution function is as:

$$F(x_q) = p. \qquad (9)$$

3.3. Reliability properties

The overall measure of consistency is reliability. If a measure produces similar results under the consistent conditions, then it is viewed that it has a high reliability. From a one testing moment to another the highly reliable scores are reproducible, accurate, and consistent. For example, measurements of the height of people's and often the weight is reliable extremely.
Reliability properties are as follows.

3.3.1. Reliability function / survival function

The probability on which a method, a patient, or further objects of attention will live outside a stated time is known as the survival function. Reliability function is another name of survival function.

On the interval $(0, 1)$with CDF $F(t)$, let T be a continuous r.v. then its reliability function for a X random variable is as:

$$R(t) = S(t) = P(T > t) = \int_t^1 f(x)dx = 1 - F(t) \qquad (10)$$

3.4. Failure Rate Function (HRF) / hazard rate function

The frequency through which a caused system or the factor fails, stated in a failure per unit of time is called a Failure rate. It is used highly in the field of engineering to check reliability and it is denoted by a Greek letter λ (lambda). Usually, the systems failure rate depends on a time, which varying upon the systems life cycle with the rate. There are a wide range of applications of Failure rates, from these applications it is noted that Failure rates are the important factors in the fields of finance, insurance, regulatory industries and in commerce.

The HRF is defined as:

$$h(t) = \frac{f(t)}{1-F(t)} = \frac{f(t)}{R(t)}. \qquad (11)$$

Where T is the failure time.

3.5. Cumulative Hazard Rate (CHR) / reversed hazard rate function

The CHR function is not included in a probability as the Hazard rate is. In a series system, the Hazard rates have a greater affinity, then for the study of a parallel systems the reversed hazard rates looks more appropriate. The function $H(t)$ is known as an integrated hazard function or a Cumulative hazard function. Obviously, also it is known as a measure of a risk. The larger value of $H(t)$ provides the larger value of a risk of failure by a time T. The CHR $H(t)$and RHR is defined as:

$$H(t) = \int_0^t h(t)dt = - \ln R\,(t). \tag{12}$$

3.6. Mean Residual Life (MRL) function

The function which measures the predictable lasting lifetime of an individual of age x at a given time x is called as MRL. If X is a non-negative random variable which representing a life of the component with distribution function F, then a MRL of variable X is denoted by $M(x)$ and defined as:

$$M(x) = \frac{1}{R(x)} \int_x^1 (y - x) f(x)\, dx. \tag{13}$$

3.7. Mean Waiting Time (MWT) function

Another important function is the MWT function, which is also called as the function of an expected idleness time.
MWT of an item failed in an interval $(0, x)$ is defined as:

$$\bar{\omega}(x) = x - \left\{ \frac{1}{F(x)} \int_0^x x f(x) dx \right\} \tag{14}$$

4. Statistical functions

The different statistical functions of a distribution are as:

4.1. Moment Generating Function (MGF)

MGF of real-valued arbitrary variable is an alternate description of its probability distribution. Hence, it provides basis of another route to an analytical result which compared along the working directly, along with a PDF's or with the CDF's.

The MGF for a X r.v. is defined as:

$$M_x(t) = E(e^{tx}). \tag{15}$$

4.2. Characteristic Function (CF)

CF fully defines the probability distribution of a r.v. which is real-valued. The CF is the Fourier transformation of PDF if a random variable admits a PDF.

The CF of a X random variable is calculated by replacing it in place of t in Equation (15) as:

$$\phi_x(t) = E(e^{itx}). \quad (16)$$

where i is an imaginary unit.

4.3. Probability Generating Function (PGF)

An illustration of a power series of the probability mass function of an arbitrary variable is known as the PGF.

PGF of a r.v. X is calculated by replacing $ln\,\alpha$ in place of t in Equation (15) as:

$$G(z) = E(z^{X\,ln\,\alpha}). \quad (17)$$

4.4. Factorial Moment Generating Function (FMGF)

The FMGF of a real-valued r.v. X is calculated by putting $ln(1+\delta)$ in place of t in equation (15) as:

$$M_x(t) = E(e^{X\,ln(1+\delta)}). \quad (18)$$

For all complex numbers t for which this expected value exists.

5. Entropies

Generally, entropy refers to a disorder or an improbability. In additional work, the entropy of an arbitrary variable X is degree of an indefinite quantity of evidence in a function. Entropies are widely used in engineering, in science, in a reliability theory and in a various situation used as an uncertainty measure. There are many entropies which have been compared and discussed in the literature but, here the expressions of most important entropies which are discussed are three as: the Shannon's entropy, the Re'nyi entropy and the β -entropy as:

5.1. Shannon's entropy

The Shannon entropy was introduced in the paper of 1948 "a mathematical theory of communication" by Claude E. Shannon. To measure the heaviness of tails and to compare the shapes of the various densities Shannon entropy plays a same role as plays by the measure of kurtosis.

The Shannon's entropy of a X r.v. is defined as:

$$Q(x) = -\int_0^1 f(x)\,log\,f\,(x)dx. \quad (19)$$

5.2 Re'nyi entropy

The Rényi entropy was developed by Alfre'd Re'nyi in 1961. This entropy is very popular in the ecology and statistics as an index of diversity. However, the Re'nyi entropy is the generalization of a Shannon's entropy.

The Re'nyi entropy for a X r.v. is denoted by$L_g(\upsilon)$ and defined as:

$$L_g(\upsilon) = \frac{1}{(1-\upsilon)} log\left\{\int_0^1 f^{\upsilon}(x)dx\right\}. \tag{20}$$

Where $\upsilon > 0$, and $\upsilon \neq 1$.

5.3. β -entropy

Havrda and Charvat introduced the β -entropy in 1967 and after that, Tsallis applied it to on the physical problems in 1988. Another name of β -entropy is known as "Tsallis entropy". With the reference of Ullah (1996) the β -entropy is also a monotonic function of the Re'nyi entropy.

For a r,v. X , the β -entropy is defined as:

$$L_\beta(\pi) = \frac{1}{(\pi-1)}\left\{1 - \int_0^1 f^{\pi}(x)dx\right\}. \tag{21}$$

Where $\pi > 0$, and $\pi \neq 0$.

6. Measures of inequality

The inequality of income metrics only has not their uses in poverty and income, also it is applicable in the other fields as in demography, in medicine, in an insurance and in a reliability. For measuring the income distribution, the social scientists are also used these inequality measures and in a specific economy the income inequality between the participants is used, in general as in the specific country or in the world. The most important income inequality metrics are as:

6.1. Gini coefficient (index)

The Gini coefficient is a degree of dispersion which denotes the distribution of wealth or income of a state's residents. However, it is used most frequently in the measure of inequalities. It is also sometimes known as a "Gini coefficient" or a "Gini ratio". This coefficient developed by a sociologist and an Italian statistician "Corrado Gini" and in 1912 he was published in his paper "Variability and Mutability". Gini coefficient for a X r.v. is:

$$G = \frac{1}{E(X)}\int_0^1 F(x)\{1 - f(x)\}d\,x. \tag{22}$$

6.2. Lorenz curve

Graphical depiction of the distribution of wealth or income is known as the Lorenz curve. Max O. Lorenz was introduced a Lorenz curve in 1905 for the depiction of the inequality of distribution of wealth.

Lorenz curve for a X r.v. is: $L(p) = \frac{1}{E(X)} \int_0^x x f(x) dx.$ (23)

6.3. Bonferroni Curve (BC)

In 1930, Bonferroni was introduced a measure of an inequality of income, which depends on the partial means, and is applicable when an important source of an inequality of income is the existence of the items when their income is below much than those of others.

BC for a X r.v. is computed by the following relation as:

$$BC(p) = \frac{L(p)}{F(x)}. \quad (24)$$

7. Fundamental properties and MGF of a 3-component mixture of power distributions

For an efficient modelling of a given time-to-failure data we plan to have the classical analysis of a 3-Component mixture of Power distribution with its different properties under the definition of an Equation (1). A finite 3-Component mixture of power distribution with mixing proportions$w_1$and$w_2$has the Probability Density Function (PDF) and Cumulative Distribution Function (CDF) as:

$$f(x;\Psi) = w_1 f_1(x;\beta_1) + w_2 f_2(x;\beta_2) + (1 - w_1 - w_2) f_3(x;\beta_3), \quad (25)$$

Where $w_1, w_2 \geq 0, w_1 + w_2 \leq 1$

$$F(x;\Psi) = w_1 F_1(x;\beta_1) + w_2 F_2(x;\beta_2) + (1 - w_1 - w_2) F_3(x;\beta_3), \quad (26)$$

Where $\Psi = (\beta_1, \beta_2, \beta_3, w_1, w_2)$

$$f_i(x;\beta_i) = \beta_i x^{\beta_i - 1}, \;\; 0 \leq x \leq 1, \beta_i > 0, i = 1, 2, 3,$$

$$f_i(x;\Psi) = w_1 \beta_1 x^{\beta_1 - 1} + w_2 \beta_2 x^{\beta_2 - 1} + (1 - w_1 - w_2) \beta_3 x^{\beta_3 - 1},$$

Where $0 \leq x \leq 1, \beta_i > 0, i = 1,2,3.$

The CDF of the i^{th} component density is given by:

$$F_i(x;\beta_i) = 1 - x^{\beta_i}, \;\; 0 \leq x \leq 1, \beta_i > 0, i = 1, 2, 3, \quad (27)$$

$$F_i(x;\Psi) = 1 - w_1 x^{\beta_1} - w_2 x^{\beta_2} - (1 - w_1 - w_2) x^{\beta_3}$$

7.1. Statistical properties of the 3-CMPD

Here, we have discussed the computable representations of some statistical properties associated with the 3-Component mixture of power distribution having PDF given in Equation (26).

7.1.1. Mean

Mean of a 3-CMPD are computed as:

$$E(X) = w_1 \int_0^1 x f_1(x;\ \beta_1)\, dx + w_2 \int_0^1 x f_2(x;\ \beta_2)\, dx + (1 - w_1 - w_2) \int_0^1 x\, f_3(x;\ \beta_3) d \quad (28)$$

After explanation, mean of a 3-CMPD becomes:

$$E(X) = \frac{w_1 \beta_1}{\beta_1 + 1} + \frac{w_2 \beta_2}{\beta_2 + 1} + \frac{(1 - w_1 - w_2)\beta_3}{\beta_3 + 1}\,. \quad (29)$$

7.1.2. r^{th} moments about origin

The r^{th} moments about origin of a 3-CMPD for a X r.v. are derived as:

$$E(X^r) = w_1 \int_0^1 x^r f_1(x;\beta_1)dx + w_2 \int_0^1 x^r f_2(x;\beta_2)dx + (1 - w_1 - w_2) \int_0^1 x f_3(x^r;\beta_3)dx$$

7.2. Variance

Variance of 3-CMPD is:

$$Var(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = w_1 \int_0^1 x^2 f_1(x;\beta_1)dx + w_2 \int_0^1 x^2 f_2(x;\beta_2)dx + (1 - w_1 - w_2)\ \int_0^1 x^2 f_3(x;\beta_3)dx \quad (30)$$

After simplification, the result of Equation (30) becomes as:

$$E(X^2) = \frac{w_1 \beta_1}{\beta_1 + 2} + \frac{w_2 \beta_2}{\beta_2 + 2} + \frac{(1 - w_1 - w_2)\beta_3}{\beta_3 + 2}\,, \quad (31)$$

After substituting the values of equations (29) and (31), in (4) the result of variance is as:

$$Var(X) = \frac{w_1 \beta_1}{\beta_1 + 2} + \frac{w_2 \beta_2}{\beta_2 + 2} + \frac{(1 - w_1 - w_2)\beta_3}{\beta_3 + 2} - \left[\frac{w_1 \beta_1}{\beta_1 + 1} + \frac{w_2 \beta_2}{\beta_2 + 1} + \frac{(1 - w_1 - w_2)\beta_3}{\beta_3 + 1}\right]^2 \quad (32)$$

7.3. h^{th} order negative moments

The h^{th}order negative moments are readily evaluated by replacing r with $(-h)$ in equation (30) as given below:

$$E(X^{-h}) = w_1 \int_0^1 x^{-h} f_1(x;\beta_1)dx + w_2 \int_0^1 x^{-h} f_2(x;\beta_2)dx + (1 - w_1 - w_2)\,,$$

$$\int_0^1 x^{-h} f_3(x;\beta_3)dx \quad (33)$$

After simplification, the result of h^{th}order negative moments are as:

$$E(X^{-h}) = \frac{w_1\beta_1}{\beta_1-h} + \frac{w_2\beta_2}{\beta_2-h} + \frac{(1-w_1-w_2)\beta_3}{\beta_3-h}. \quad (34)$$

7.4. Quantile function

The quantile function for a X r.v. is:

$$x_{q=} w_1 x^{\beta_1} + w_2 x^{\beta_2} + (1-w_1-w_2)x^{\beta_3}. \quad (35)$$

The median Q_2 from equation (41) are as:

$$\text{Median} = w_1(0.50)^{\beta_1} + w_2(0.50)^{\beta_2} + (1-w_1-w_2)(0.50)^{\beta_3}. \quad (36)$$

The quartile Q_1, $Q_3$and Bowley's coefficient of skewness from Equation (35) are as:

$$x_{0.25} = Q_1 = w_1(0.25)^{\beta_1} + w_2(0.25)^{\beta_2} + (1-w_1-w_2)(0.25)^{\beta_3}. \quad (37)$$

$$x_{0.75} = Q_3 = w_1(0.75)^{\beta_1} + w_2(0.75)^{\beta_2} + (1-w_1-w_2)(0.75)^{\beta_3}. \quad (38)$$

And

$$Skewness = \frac{Q_3+Q_1-2Q_2}{Q_3-Q_1}. \quad (39)$$

By putting the results of equations (36), (37) and (38) in (39) the skewness of a 3-component mixture of Power distributions can be easily calculated.

Similarly, by using the expression of equations (29), (36), (37) and (39) the numerical results of mean, median, mode, variance and skewness of a 3-CMPD are obtained and are presented in Table-1.

Table-1: Quantile Function of a 3-Component Mixture of Power Distributions

$\beta_1, \beta_2, \beta_3, w_1, w_2$	Mean	Variance	Median	Mode	Skewness
2,3,4,0.1,0.2	0.776667	0.0334556	0.822829	0.95176	-0.757145
2,3,4,0.2,0.3	0.758333	0.038239	0.80701	0.91885	-0.746535
2,3,4,0.3,0.4	0.74234	0.043214	0.789285	0.89280	-0.71805
2,3,4,0.4,0.5	0.721667	0.045863	0.769657	0.86268	-0.672257
4,3,2,0.2,0.1	0.701667	0.050997	0.750502	0.83907	-0.648754
5,4,3,0.3,0.2	0.78512	0.031391	0.830462	0.86759	-0.769745
6,5,4,0.4,0.3	0.832857	0.020634	0.871803	0.94142	-0.813368
7,6,5,0.5,0.4	0.86369	0.014356	0.896984	0.923345	-0.833615
1,2,3,0.4,0.2	0.633333	0.072222	0.687893	0.81286	-0.609056
3,2,1,0.4,0.2	0.633333	0.072222	0.687893	0.83464	-0.609056
5,4,3,0.4,0.2	0.793333	0.029669	0.838294	0.85668	-0.783057
7,6,5,0.4,0.2	0.854762	0.016207	0.890095	0.98425	-0.832623

The statement refers to the skewness of the Three-Component Mixture of Power Distributions (3-CMPD). Skewness is a measure of asymmetry in the distribution. In a negatively skewed (or left-skewed) distribution, the tail on the left side is longer, and most of the data is concentrated on the right.

The entry highlights that, for the 3-CMPD, the Mean < Median < Mode, which is a characteristic of a negatively skewed distribution.

8. Reliability properties of the 3-CMPD

The reliability properties of a 3-CMPD are:

8.1. Reliability Function (RF)

The RF are evaluated by this formula as:

$$R(x;\Psi) = w_1 R_1(x;\beta_1) + w_2 R_2(x;\beta_2) + (1 - w_1 - w_2) R_3(x;\beta_3)\ , \tag{40}$$

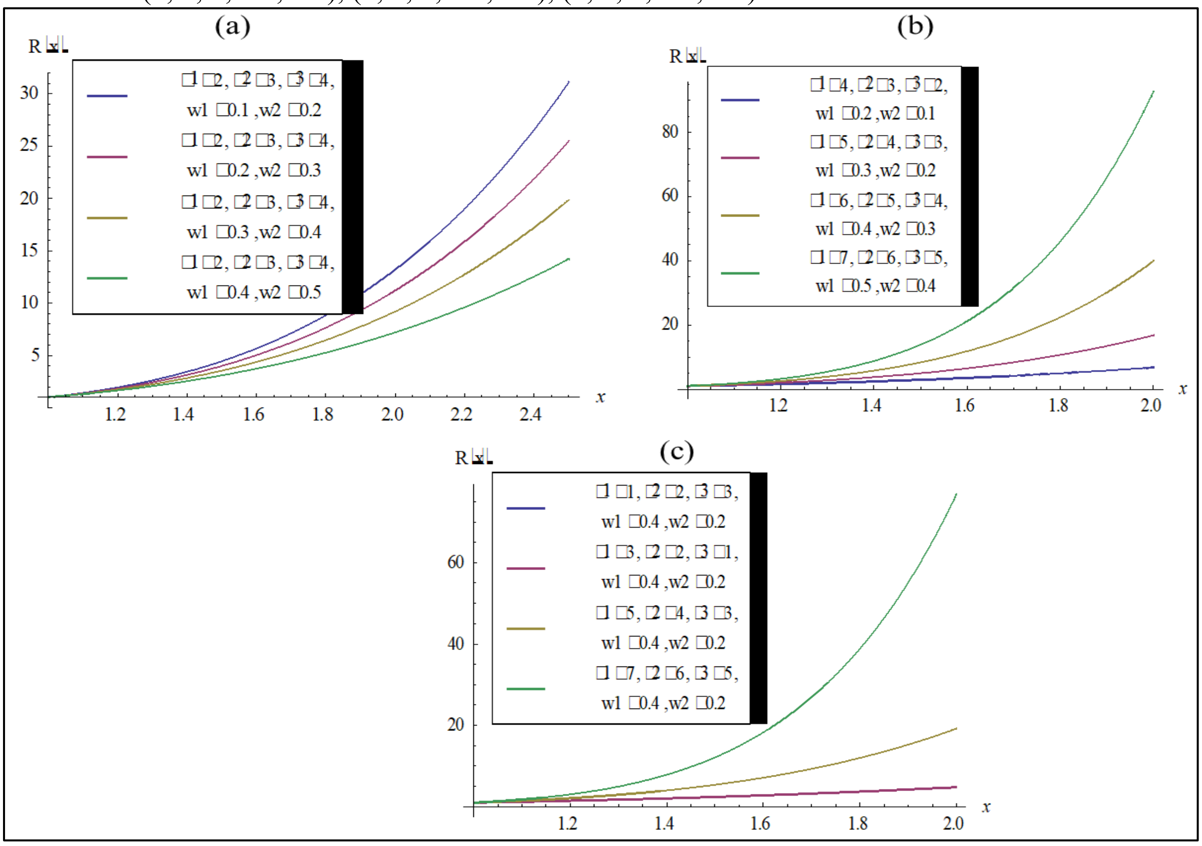
Where $R_i(x;\beta_i)$ is the RF of the i^{th}component as:

$R_i(x;\beta_i) = x^{\beta_i}, 0 \leq x \leq 1, \beta_i > 0, i = 1, 2, 3.$

After simplification, RF is as:

$$\mathrm{R}(x;\Psi) = w_1 x^{\beta_1} + w_2 x^{\beta_2} + (1 - w_1 - w_2) x^{\beta_3} \tag{41}$$

Figure 2(a): Graphs of RF for different parametric values (β_1,β_2,β_3,w_1,w_2) = (2, 3, 4, 0.1, 0.2), (2, 3, 4, 0.2, 0.3), (2, 3, 4, 0.3, 0.4), (2, 3, 4, 0.4, 0.5);
Figure 2(b): Graphs of RF for different parametric values (β_1,β_2,β_3,w_1,w_2) = (4, 3, 2, 0.2, 0.1), (5, 4, 3, 0.3, 0.2), (6, 5, 4, 0.4, 0.3), (7, 6, 5, 0.5, 0.4);
Figure 2(c): Graphs of RF for different parametric values (β_1,β_2,β_3,w_1,w_2) = (1, 2, 3, 0.4, 0.2), (3, 2, 1, 0.4, 0.2), (5, 4, 3, 0.4, 0.2), (7, 6, 5, 0.4, 0.2).



For some of the fixed values of component and the proportions parameter the behaviour of RF for the 3-CMPD is shown in Figure 2. The effect of component and proportion parameters $\beta_1, \beta_2, \beta_3$, w_1 and w_2 on the RF for the 3-CMPD can be observed from the Figures 4.1-4.3. The flexibility of the RF for 3-CMPD is also explain by these graphs.

The parametric values of the RF of a 3-CMPD fixed in the Figures 2 are evaluated by using the expression in Equation (45). So, the numerical results of the reliability function or survival function, are obtained and are presented in Table-2.

Table-2: Reliability Function of a 3-Component Mixture of Power Distributions

$\beta_1, \beta_2, \beta_3, w_1, w_2$	X=1	X=2	X=3	X=5	X=7
2,3,4,0.1,0.2	1	13.2	63	465	1754.2
2,3,4,0.2,0.3	1	11.2	50.4	355	1313.2
2,3,4,0.3,0.4	1	9.2	37.8	245	872.2
2,3,4,0.4,0.5	1	7.2	25.2	135	431.2
4,3,2,0.2,0.1	1	6.8	25.2	155	548.8
5,4,3,0.3,0.2	1	16.8	102.6	1125	5693.8
6,5,4,0.4,0.3	1	40	388.8	7375	52822
7,6,5,0.5,0.4	1	92.8	1409.4	45625	460512
3,2,1,0.4,0.2	1	4.8	13.8	57	149.8
1,2,3,0.4,0.2	1	4.8	13.8	57	149.8
5,4,3,0.4,0.2	1	19.2	124.2	1425	7340.2
7,6,5,0.4,0.2	1	76.8	1117.8	35625	359670

From the Figure 2 and the entries in Table-2, in general, it is shown that the RF for a 3-CMPD follows a decreasing trend over the time. In row 1 of the Table-2 when $\beta_1 < \beta_2 < \beta_3$ the reliability function or the survival function decreases as the proportion parameters increase. On the other hand, in row 2 and 3 of the Table-2 when $\beta_1 > \beta_2 > \beta_3$ the RF increases as component parameters increase.

8.2. Hazard rate function

The HRF for a X r.v. is defined as:

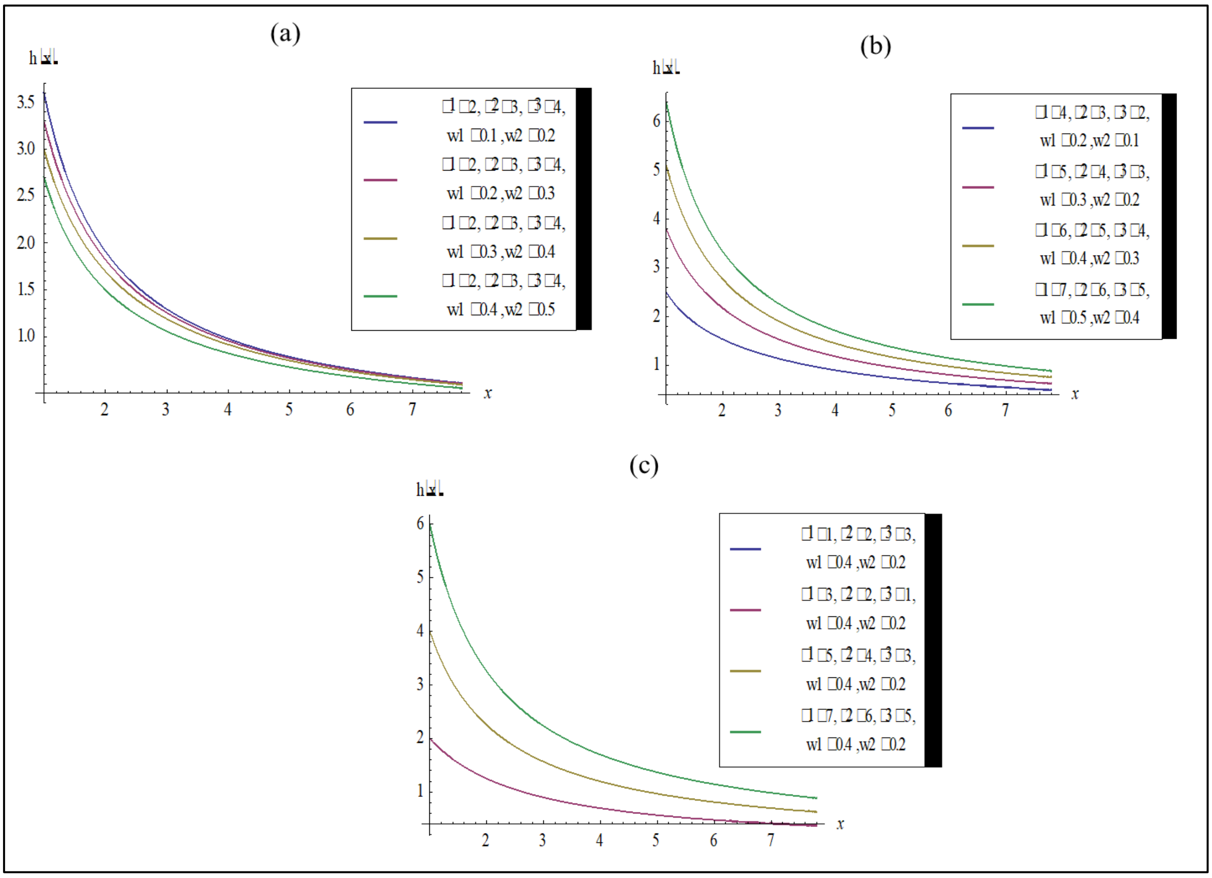
$$h(x;\Psi) = \frac{f(x;\Psi)}{R(x;\Psi)} = \frac{w_1 f_1(x;\beta_1) + w_2 f_2(x;\beta_2) + (1-w_1-w_2) f_3(x;\beta_3)}{w_1 R_1(x;\beta_1) + w_2 R_2(x;\beta_2) + (1-w_1-w_2) R_3(x;\beta_3)},$$

By putting the values of density function.

$$\mathrm{h}(x;\Psi) = \frac{w_1 \beta_1 x^{\beta_1 - 1} + w_2 \beta_2 x^{\beta_2 - 1} + (1-w_1-w_2)\beta_3 x^{\beta_3 - 1}}{w_1 x^{\beta_1} + w_2 x^{\beta_2} + (1-w_1-w_2) x^{\beta_3}}.$$

For some of the fixed values of component and the proportion parameters, the behaviour of the HRF for 3-CMPD is shown in Figure 3. The effect of component and proportion parameters $\beta_1, \beta_2, \beta_3$, w_1 and $\boldsymbol{w_2}$ on the HRF for the 3-CMPD can be observed from the Figure 3. The flexibility of the HRF for the 3-CMPD is also explain by these graphs.

Figure 3(a): Graphs of HRF for different parametric values ($\beta_1, \beta_2, \beta_3, w_1, w_2$) = (2, 3, 4, 0.1, 0.2), (2, 3, 4, 0.2, 0.3), (2, 3, 4, 0.3, 0.4), (2, 3, 4, 0.4, 0.5).
Figure 3(b): Graphs of HRF for different parametric values ($\beta_1, \beta_2, \beta_3, w_1, w_2$) = (4, 3, 2, 0.2, 0.1), (5, 4, 3, 0.3, 0.2), (6, 5, 4, 0.4, 0.3), (7, 6, 5, 0.5, 0.4).
Figure 3(c): Graphs of HRF for different parametric values ($\beta_1, \beta_2, \beta_3, w_1, w_2$) = (1, 2, 3, 0.4, 0.2), (3, 2, 1, 0.4, 0.2), (5, 4, 3, 0.4, 0.2), (7, 6, 5, 0.4, 0.2).



The parametric values of the HRF of a 3-CMPD which fixed in the Figure 3 are evaluated by using the expression in an Equation (46). So, the numerical results of the HRF, are obtained and are presented in Table-3.

Table-3: HR Function of a 3-Component Mixture of Power Distributions

$\beta_1, \beta_2, \beta_3, w_1, w_2$	X=1	X=2	X=3	X=5	X=7
2,3,4,0.1,0.2	3.6	1.90909	1.29524	0.787097	0.565044
2,3,4,0.2,0.3	3.3	1.82143	1.25595	0.773239	0.558102
2,3,4,0.3,0.4	3	1.69565	1.19048	0.746939	0.544141
2,3,4,0.4,0.5	2.7	1.5	1.05952	0.677778	0.501623
4,3,2,0.2,0.1	2.5	1.52941	1.13095	0.73871	0.544643
5,4,3,0.3,0.2	3.8	2.16667	1.52632	0.955556	0.693632
6,5,4,0.4,0.3	5.1	2.76	1.89583	1.16441	0.83961
7,6,5,0.5,0.4	6.4	3.32759	2.25287	1.36986	0.984359
1,2,3,0.4,0.2	2	1.25	0.898551	0.568421	0.413885
3,2,1,0.4,0.2	2	1.25	0.898551	0.568421	0.413885
5,4,3,0.4,0.2	4	2.25	1.56522	0.968421	0.699599
7,6,5,0.4,0.2	6	3.25	2.23188	1.36842	0.985314

From the Figures 3 and the entries in Table-3, in general, it is shown that the HRF for a 3-CMPD follows an increasing trend over the time. In row 1 of the Table-3 when $\beta_1 < \beta_2 < \beta_3$ the HRF increases as the proportion parameters increase. On the other hand, in row 2 and 3 of the Table-3 when $\beta_1 > \beta_2 > \beta_3$ the HRF decreases when the component parameter increases.

8.3. CHR and reversed hazard rate function

The CHR $\mathrm{H}(x;\Psi)$ and RHRF $r(x;\Psi)$ of a 3-CMPD for a X r.v. is defined as:

$$H(x;\Psi_i) = \int_0^x h(u;\Psi_i)du = -\ln R(x;\Psi_i),$$

$$\mathrm{H}(x;\Psi) = -\ln\{w_1 x^{\beta_1} + w_2 x^{\beta_2} + (1-w_1-w_2)x^{\beta_3}\}, \quad (42)$$

Also

$$r(x;\Psi) = \frac{f(x;\Psi)}{F(x;\Psi)} = \frac{w_1 f_1(x;\beta_1)+w_2 f_2(x;\beta_2)+(1-w_1-w_2)f_3(x;\beta_3)}{w_1 F_1(x;\beta_1)+w_2 F_2(x;\beta_2)+(1-w_1-w_2)F_3(x;\beta_3)},$$

$$r(x;\Psi) = \frac{w_1\beta_1 x^{\beta_1-1}+w_2\beta_2 x^{\beta_2-1}+(1-w_1-w_2)\beta_3 x^{\beta_3-1}}{1-w_1 x^{\beta_1}-w_2 x^{\beta_2}-(1-w_1-w_2)x^{\beta_3}}. \quad (43)$$

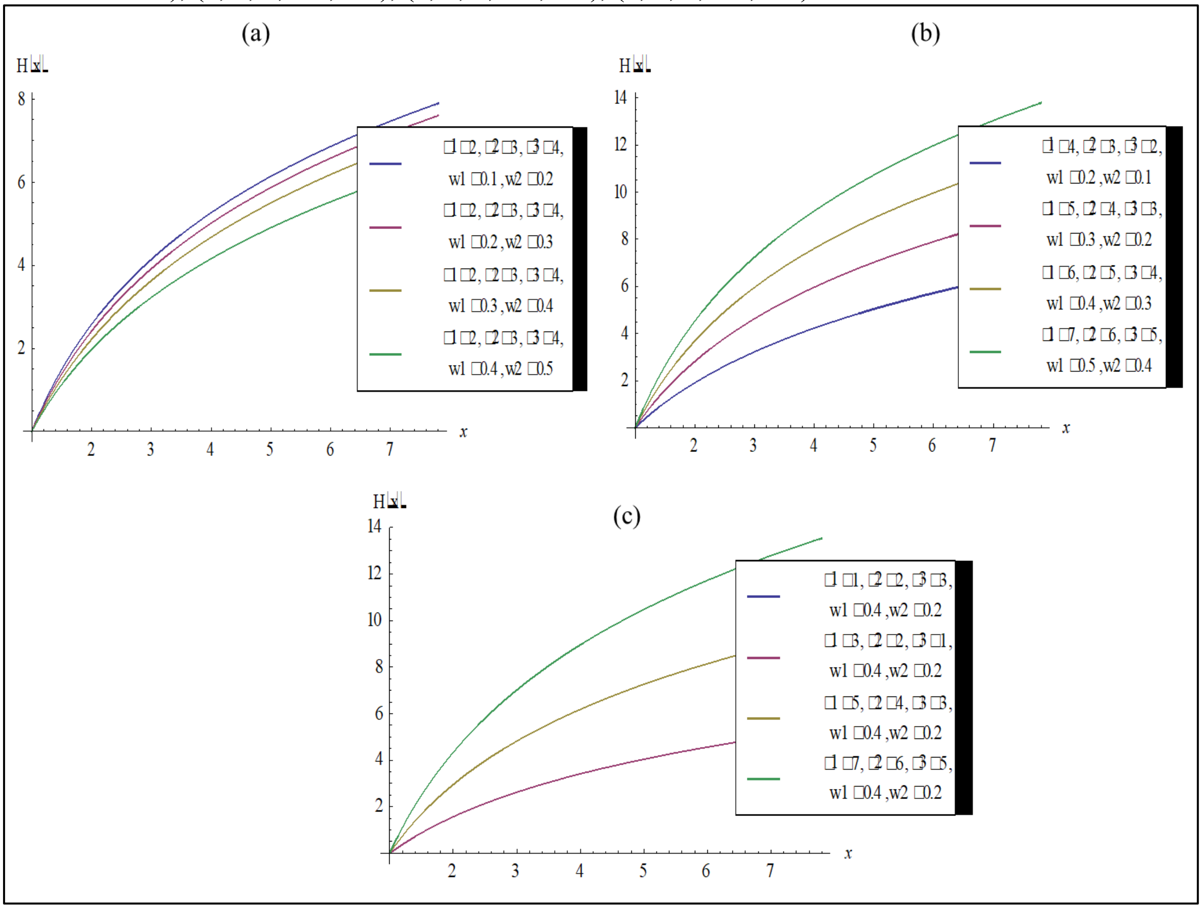
For some of the fixed values of component and the proportion parameters the behaviour of the CHR for the 3-CMPD is shown in Figure 4. The effect of component and proportion parameters $\beta_1, \beta_2, \beta_3$, w_1 and w_2 on the CHR for the 3-CMPD can be observed from the Figure 4. The flexibility of the CHR for the 3-CMPD is also explain by these graphs.

Table-4: Cumulative Rate Function of a 3-Component Mixture of Power Distributions

$\beta_1, \beta_2, \beta_3, w_1, w_2$	X=1	X=2	X=3	X=5	X=7
2,3,4,0.1,0.2	0.277778	0.52381	0.772059	1.27049	1.76977
2,3,4,0.2,0.3	0.30303	0.54902	0.796209	1.29326	1.79179
2,3,4,0.3,0.4	0.333333	0.333333	0.84000	1.3388	1.83776
2,3,4,0.4,0.5	0.37037	0.666667	0.94382	1.47541	1.99353
4,3,2,0.2,0.1	0.40000	0.653846	0.884211	1.35371	1.83607
5,4,3,0.3,0.2	0.263158	0.461538	0.655172	1.04651	1.44169
6,5,4,0.4,0.3	0.196078	0.362319	0.527473	0.858806	1.19103
7,6,5,0.5,0.4	0.15625	0.300518	0.443878	0.73	1.01589
1,2,3,0.4,0.2	0.5000	0.8000	1.1129	1.75926	2.41613
3,2,1,0.4,0.2	0.5000	0.8000	1.1129	1.75926	2.41613
5,4,3,0.4,0.2	0.2500	0.444444	0.638889	1.03261	1.42939
7,6,5,0.4,0.2	0.166667	0.307692	0.448052	0.730769	1.01491

From the Figure 4 and the entries in the Table-4, in general, it is shown that the CHR for a 3-CMPD follows an increasing trend over the time. In the row 1 of the Table-4, when the $\beta_1 < \beta_2 < \beta_3$ the CHR increases as the proportion parameters increase. On the other hand, in the row 2 and 3 of the Table-4 when $\beta_1 > \beta_2 > \beta_3$ the CHR decreases when the component parameter increases.

Figure 4(a): Graphs of CHR function for different parametric values ($\beta_1, \beta_2, \beta_3, w_1, w_2$) = (2, 3, 4, 0.1, 0.2), (2, 3, 4, 0.2, 0.3), (2, 3, 4, 0.3, 0.4), (2, 3, 4, 0.4, 0.5).
Figure 4(b): Graphs of CHR function for different parametric values ($\beta_1, \beta_2, \beta_3, w_1, w_2$) = (4, 3, 2, 0.2, 0.1), (5, 4, 3, 0.3, 0.2), (6, 5, 4, 0.4, 0.3), (7, 6, 5, 0.5, 0.4).
Figure 4(c): Graphs of CHR function for different parametric values ($\beta_1, \beta_2, \beta_3, w_1, w_2$) = (1, 2, 3, 0.4, 0.2), (3, 2, 1, 0.4, 0.2), (5, 4, 3, 0.4, 0.2), (7, 6, 5, 0.4, 0.2).



The parametric values of the cumulative rate function of a 3-CMPD which fixed in the Figure 4 are evaluated by using the expression in an Equation (47). So, the numerical results of the CHR, are obtained and are presented in Table-4.

8.4. MRL function

MRL function for a X r.v. is defined as:

$$m(x;\Psi) = \frac{1}{R(x;\Psi)} \int_x^1 u f(x;\Psi) du - x \,, \tag{44}$$

$$m(x;\Psi) = \frac{1}{R(x;\Psi)} \left\{ E(x) - \int_0^x u f(x;\Psi) du \right\} - x \,, \tag{45}$$

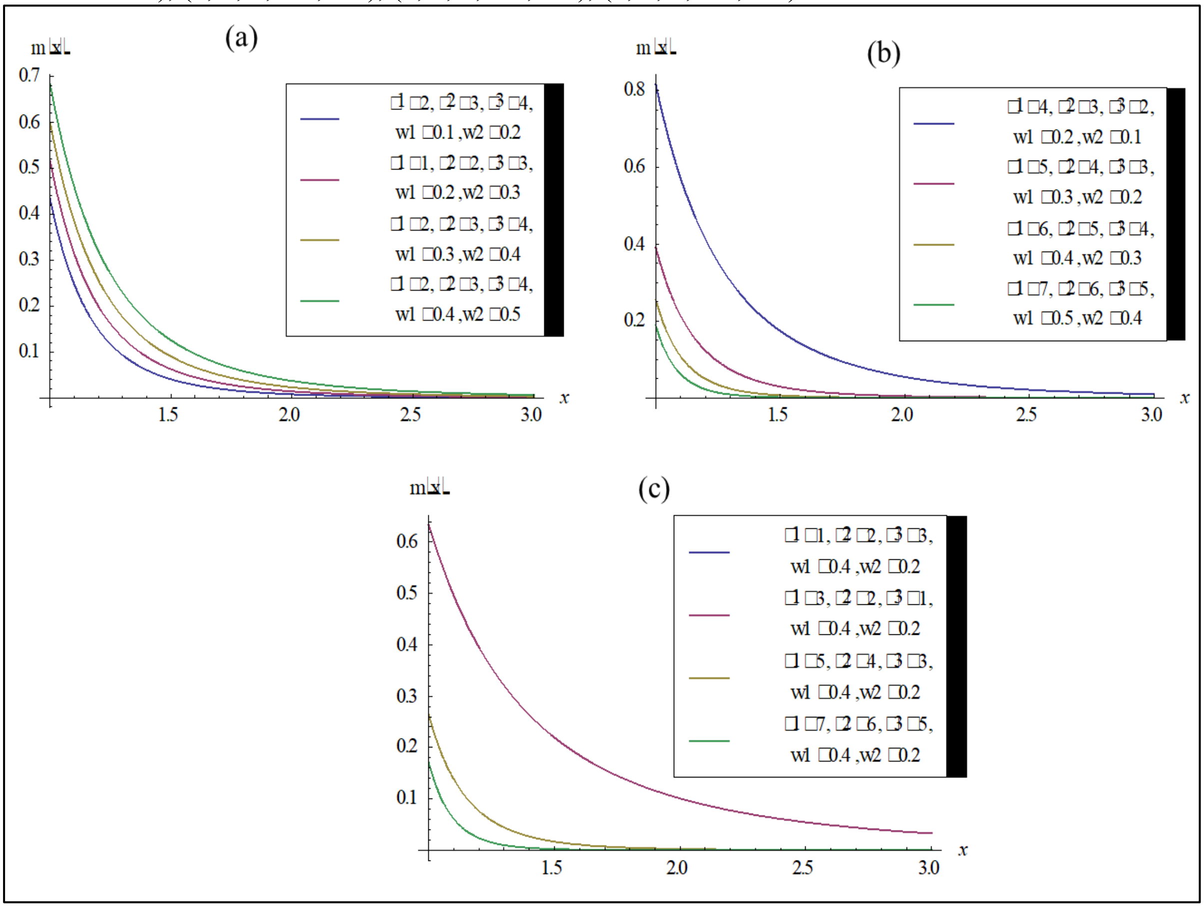
After solving the integral $\int_0^x u f(x;\Psi) du$ the value is

$$m(x;\Psi) = \frac{1}{w_1 x^{\beta_1} - w_2 x^{\beta_2} - (1-w_1-w_2) x^{\beta_3}} \left\{ \frac{w_1\beta_1}{\beta_1+1} + \frac{w_2\beta_2}{\beta_2+1} + \frac{(1-w_1-w_2)\beta_3}{\beta_3+1} - \int_x^1 u f(x;\Psi) du \right\} - x, \tag{46}$$

$$m(x;\Psi) = \frac{\frac{w_1}{\beta_1-1}x^{1-\beta_1} + \frac{w_2}{\beta_2-1}x^{1-\beta_2} + \frac{1-w_1-w_2}{\beta_3-1}x^{1-\beta_3}}{w_1x^{\beta_1}+w_2x^{\beta_2}+(1-w_1-w_2)x^{\beta_3}}. \quad (47)$$

For some of the fixed values of component and the proportion parameters the behaviour of the MRL function for the 3-CMPD is shown in Figure 5. The effect of parameters $\beta_1, \beta_2, \beta_3$, w_1 and w_2 on the MRL function for the 3-CMPD can be observed from the Figure 5. The flexibility of the MRL function for the 3-CMPD is also explain by these graphs.

Figure 5(a): Graphs of MRL function for different parametric values ($\beta_1, \beta_2, \beta_3, w_1, w_2$) = (2, 3, 4, 0.1, 0.2), (2, 3, 4, 0.2, 0.3), (2, 3, 4, 0.3, 0.4), (2, 3, 4, 0.4, 0.5).
Figure 5(b): Graphs of MRL function for different parametric values ($\beta_1, \beta_2, \beta_3, w_1, w_2$) = (4, 3, 2, 0.2, 0.1), (5, 4, 3, 0.3, 0.2), (6, 5, 4, 0.4, 0.3), (7, 6, 5, 0.5, 0.4).
Figure 5(c): Graphs of MRL function for different parametric values ($\beta_1, \beta_2, \beta_3, w_1, w_2$) = (1, 2, 3, 0.4, 0.2), (3, 2, 1, 0.4, 0.2), (5, 4, 3, 0.4, 0.2), (7, 6, 5, 0.4, 0.2).



The parametric values of the MRL function of a 3-CMPD which fixed in the Figure 5 are evaluated by using the expression in an equation (47). So, the numerical results of the MRL function, are obtained and are presented in Table-5.

From the Figures 5 and the entries in Table-5, in general, it is shown that the MRL function for a 3-CMPD follows a decreasing trend over the time. In row 1 of the Table-5 when $\beta_1 < \beta_2 < \beta_3$ the MRL function decreases as the proportion parameters increase. On the other hand, in row 2 and 3 of the Table-5 when $\beta_1 > \beta_2 > \beta_3$ the MRL function increases when the component parameter increases.

Table-5: : MRL Function of a 3-Component Mixture of Power Distributions

$\beta_1, \beta_2, \beta_3, w_1, w_2$	X=1	X=2	X=3	X=5	X=7
2,3,4,0.1,0.2	0.43333	0.007841	0.0008426	0.00005562	0.0000096
2,3,4,0.2,0.3	0.51667	0.014169	0.0017592	0.0001333	0.00004582
2,3,4,0.3,0.4	0.60000	0.023078	0.0033137	0.0002816	0.00005408
2,3,4,0.4,0.5	0.683333	0.037037	0.00644229	0.00068642	0.0001578
4,3,2,0.2,0.1	0.816667	0.0545343	0.0095777	0.00091957	0.00184429
5,4,3,0.3,0.2	0.391667	0.00449529	0.00030829	0.0000094	0.00000093
6,5,4,0.4,0.3	0.25500	0.00049188	0.00001742	0.00000012	0.00000
7,6,5,0.5,0.4	0.188333	0.00057808	0.00000053	0.000000	0.00000
1,2,3,0.4,0.2	0.633333	0.100694	0.0324745	0.00746199	0.00278376
3,2,1,0.4,0.2	0.633333	0.100694	0.0324745	0.00746199	0.00278376
5,4,3,0.4,0.2	0.263333	0.00232205	0.00014144	0.0000041	0.0000004
7,6,5,0.4,0.2	0.170476	0.00009023	0.0000010	0.000000	0.000000

8.5 MWT function

MWT function is defined as:

$$\bar{\mu}(x;\Psi) = x - \left\{\frac{1}{F(x;\Psi)}\int_0^x xf(x;\Psi)dx\right\}, \tag{48}$$

By putting the result of equations (26) and (46) in (47) the result of the MWT function is as:

$$\bar{\mu}(x;\Psi) = x - \left\{\frac{1}{1-w_1x^{\beta_1}-w_2x^{\beta_2}-(1-w_1-w_2)x^{\beta_3}}\int_0^x xf(x;\Psi)dx\right\},$$

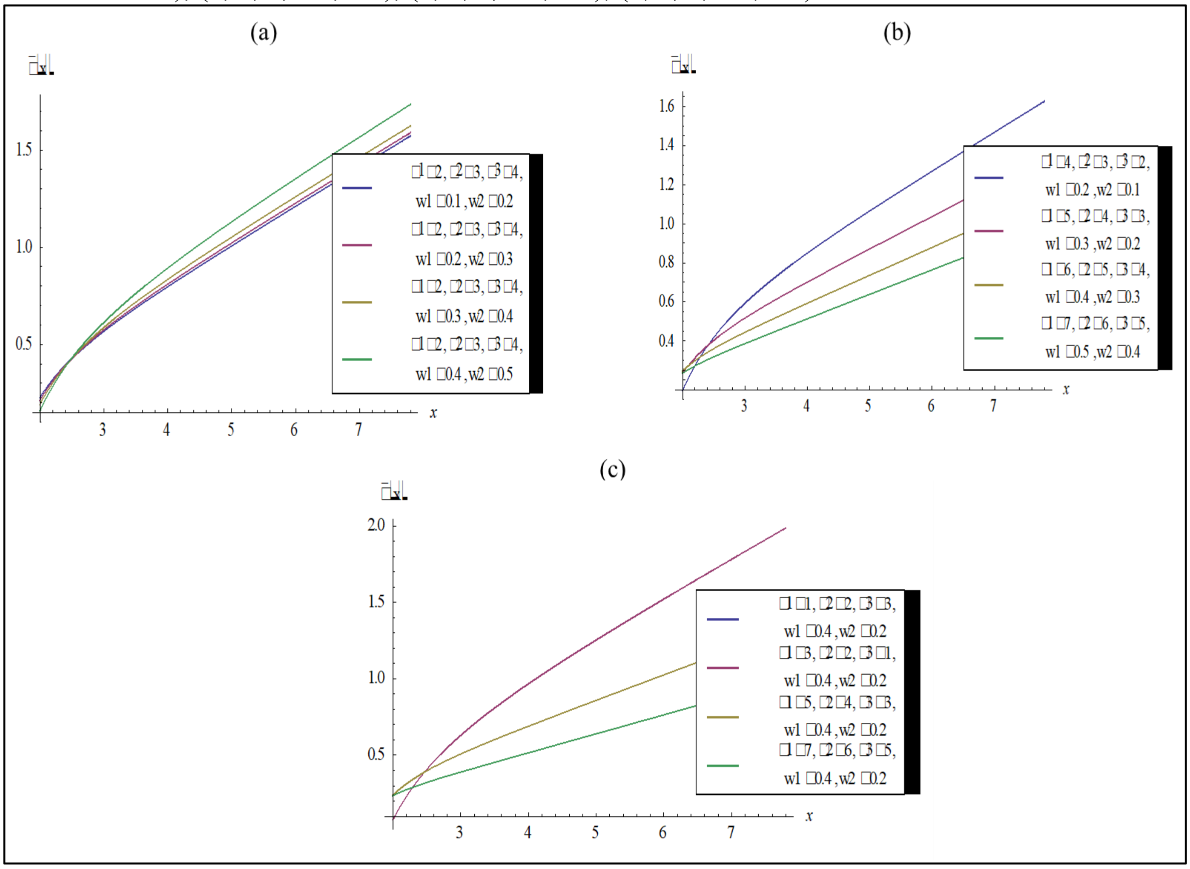
$$\bar{\mu}(x;\Psi) = \frac{x-\frac{w_1}{-\beta_1-1}[x^{\beta_1+1}-1]+\frac{w_2}{-\beta_2-1}[x^{\beta_2+1}-1]+\frac{1-w_1-w_2}{-\beta_3-1}[x^{\beta_3+1}-1]}{1-w_1x^{\beta_1}-w_2x^{\beta_2}-(1-w_1-w_2)x^{\beta_3}} \tag{49}$$

Table-6: MWT Function of a 3-Component Mixture of Power Distributions

$\beta_1, \beta_2, \beta_3, w_1, w_2$	X=1	X=2	X=3	X=5	X=7
2,3,4,0.1,0.2	0.002404	0.272404	0.576559	1.00796	1.41298
2,3,4,0.2,0.3	0.023889	0.263889	0.585695	1.02392	1.42996
2,3,4,0.3,0.4	0.03122	0.25122	0.601087	1.05426	1.46414
2,3,4,0.4,0.5	0.03037	0.230376	0.632507	1.13443	1.56839
4,3,2,0.2,0.1	0.02123	0.21523	0.609366	1.06814	1.46959
5,4,3,0.3,0.2	0.02937	0.269937	0.522441	0.871139	1.20286
6,5,4,0.4,0.3	0.06326	0.26326	0.445679	0.736082	1.02125
7,6,5,0.5,0.4	0.042316	0.242316	0.386288	0.638559	0.888825
1,2,3,0.4,0.2	0.049123	0.149123	0.651042	1.25833	1.7836
3,2,1,0.4,0.2	0.039123	0.149123	0.651042	1.25833	1.7836
5,4,3,0.4,0.2	0.021429	0.271429	0.513095	0.859522	1.19202
7,6,5,0.4,0.2	0.03451	0.2451	0.390393	0.640016	0.888612

For some of the fixed values of component and the proportion parameters the behaviour of the MWT function for the 3-CMPD is shown in Figure (6). The effect of component and proportion parameters $\beta_1, \beta_2, \beta_3$, w_1 and w_2 on the MWT function for the 3-CMPD can be observed from the Figure (6). The flexibility of the MWT function for the 3-CMPD is also explain by these graphs.

Figure 6(a): Graphs of MWT function for different parametric values $(\beta_1, \beta_2, \beta_3, w_1, w_2)$ = (2, 3, 4, 0.1, 0.2), (2, 3, 4, 0.2, 0.3), (2, 3, 4, 0.3, 0.4), (2, 3, 4, 0.4, 0.5).
Figure 6(b): Graphs of MWT function for different parametric values $(\beta_1, \beta_2, \beta_3, w_1, w_2)$ = (4, 3, 2, 0.2, 0.1), (5, 4, 3, 0.3, 0.2), (6, 5, 4, 0.4, 0.3), (7, 6, 5, 0.5, 0.4).
Figure 6(c): Graphs of MWT function for different parametric values $(\beta_1, \beta_2, \beta_3, w_1, w_2)$ = (1, 2, 3, 0.4, 0.2), (3, 2, 1, 0.4, 0.2), (5, 4, 3, 0.4, 0.2), (7, 6, 5, 0.4, 0.2).



The parametric values of the MWT function of a 3-CMPD which fixed in the Figure 6 are evaluated by using the expression in an Equation (49). So, the numerical results of the MWT function, are obtained and are presented in Table-6.

From the Figure 6 and the entries in Table-6, in general, it is shown that the MWT function for a 3-CMPD follows an increasing trend over the time. In row 1 of the Table-6 when $\beta_1 < \beta_2 < \beta_3$ the MWT function increases as the proportion parameters increase. On the other hand, in row 2 and 3 of the Table-6 when $\beta_1 > \beta_2 > \beta_3$ the MWT function decreases when the component parameter increases.

9. Statistical functions of the 3-CMPD

The different types of statistical functions of a 3-CMPD are given as:

9.1. MGF of 3-CMPD

The MGF of a 3-CMPD for a *X* r.v. is derived as:

$$M_X(t) = E(e^{tx}),$$

$$M_X(t) = w_1 \int_0^1 e^{tx} \beta_1 x^{\beta_1 - 1} dx + w_2 \int_0^1 e^{tx} \beta_2 x^{\beta_2 - 1} dx + (1 - w_1 - w_2) \int_0^1 e^{tx} \beta_3 x^{\beta_3 - 1} dx \quad (50)$$

9.2. CF of 3-CMPD

The CF of a 3-CMPD for a *X* r.v. is obtained as:

$$\varphi_X(t) = E\left(e^{itx}\right),$$

The CF of 3-CMPD is:

$$\varphi_X(t) = w_1 \int_0^1 e^{itx} \beta_1 x^{\beta_1 - 1} dx + w_2 \int_0^1 e^{itx} \beta_2 x^{\beta_2 - 1} dx + (1 - w_1 - w_2) \int_0^1 e^{itx} \beta_3 x^{\beta_3 - 1} dx \quad (51)$$

10. Conclusion

A Three-Component Mixture of Power Distributions (3-CMPD) has been introduced, with several fundamental statistical properties derived and analysed. These include key metrics such as the mean, median, variance, and quantile function. Additionally, the moments of the origin, negative moments, factorial moments, and the coefficient of skewness are discussed. Mathematical expressions for important related statistical functions have also been derived, including the reliability functions, moment generating function, characteristic function, probability generating function, and factorial moment generating function. The study further explores the expressions of three critical entropy measures: Shannon's entropy, -entropy, and Renyi's entropy. Moreover, the mathematical expressions for various inequality indexes, such as the Gini index, Lorenz curve, Bonferroni curve, Zenga index, Atkinson index, and the Generalized entropy index, are analysed.

In the context of reliability analysis, several key functions are derived and discussed: the Reliability Function (RF), Hazard Rate Function (HRF), Cumulative Hazard Rate (CHR) function, Reversed Hazard Rate (RHR) function, Mean Residual Life (MRL) function, and Mean Waiting Time (MWT) function. The study includes numerical evaluations and graphical representations to illustrate the behaviour of these reliability properties across different parameter values and component proportions. This comprehensive analysis provides a robust mathematical foundation for understanding the statistical and reliability properties of the 3-CMPD, offering valuable insights for applications where mixture distributions are relevant.

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ORCID iD:

Umar Waqas	https://orcid.org/0009-0003-0832-6734
Nazakat Ali	https://orcid.org/0009-0005-9663-1935
Hafiz Bilal Ahmad	https://orcid.org/0009-0003-1061-2174
Amir Shahzad	https://orcid.org/0009-0004-1307-468X
Suhail Ashraf	https://orcid.org/0009-0001-4302-6097

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